HD-MPC Industrial Workshop

Distributed and hierarchical MPC: Main concepts and challenges

Leuven, June 24, 2011

1. Control of large-scale systems
2. Model predictive control (MPC)
3. Distributed MPC
4. Hierarchical MPC
5. Main issues and topics in HD-MPC

Challenges in control of large-scale systems:

- Large-scale nature of the system
- Distributed vs centralized control
- Optimality $\leftrightarrow$ computational efficiency/tractability
- Global $\leftrightarrow$ local
- Scalability,
- Communication requirements (bandwidth)
- Robustness against failures

$\rightarrow$ multi-level or distributed approach

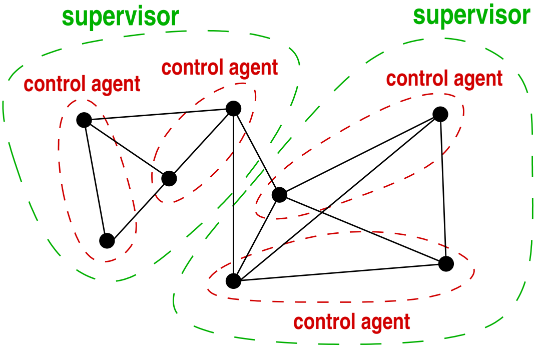
Distributed or multi-agent control

- Autonomous or semi-autonomous control agents
- Limited “view”
- Communication with neighboring agents
- Cooperation to contribute to optimal operation of total system
- Coordination to solve conflicts & to prevent counteracting

→ embed in multi-level orhierarchical framework for further scalability and coordination

Multi-level control

- Multi-level control with intelligent control agents & coordination
- Time-based and space-based separation into layers



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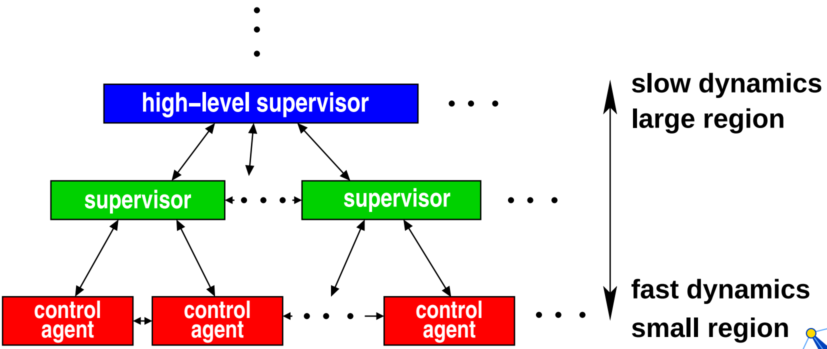
- Time-based and space-based separation into layers

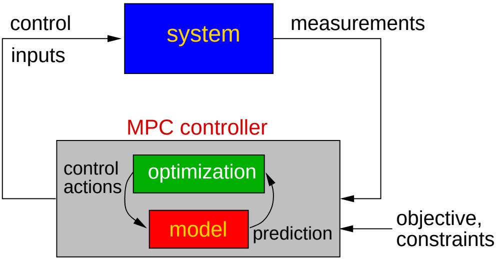


Multi-level control framework

- Lowest level:
  - local control agents
  - "fast" control
  - small region
  - operational control
- Higher levels:
  - supervisors
  - "slower" control
  - larger regions
  - operational, tactical, strategic control
- Multi-level, multi-objective control structure
- *Coordination at and across all levels required*
- Combine with model predictive control (MPC)

MPC: Principle of operation

- Performance/objective function (e.g., reference tracking versus input energy)
- Prediction model
- Constraints
- (On-line) optimization
- Receding horizon



Nonlinear optimization problem: $\min_{\mathbf{u}_k} J^{\mathrm{MPC}}_{k,N_\mathrm{p}}(\mathbf{u}_k)$

subject to system dynamics, operational constraints

where $\mathbf{u}_k = [u(k)\ u(k+1)\ \cdots\ u(k+N_\mathrm{p}-1)]^\mathrm{T}$

Major problem for MPC in practice: Required computation time for large-scale systems

- Use distributed and/or hierarchical control approach
- Choice of the prediction model: accuracy versus computational complexity
- Right optimization approach
  - parallel and/or distributed optimization
  - approximate original MPC optimization problem by another optimization problem that can be solved efficiently

Major problem for MPC in practice: Required computation time for large-scale systems

- Use *distributed* and/or *hierarchical control* approach
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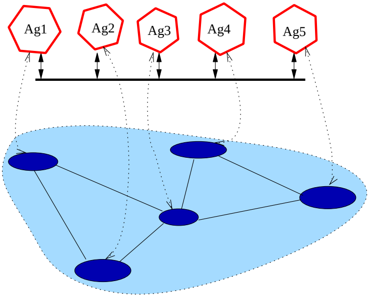
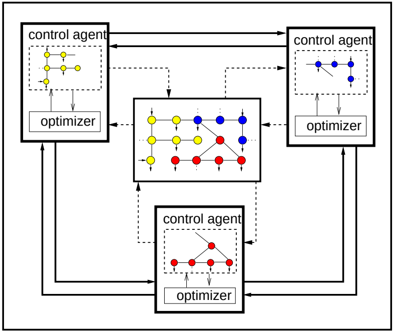
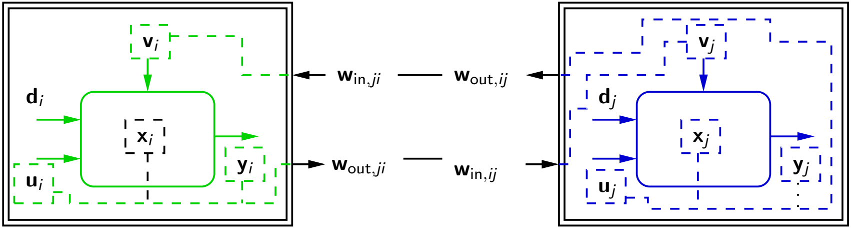
- Subsystems instead of overall system
- Single agent/controller for each subsystem
  - limited action capabilities
  - limited information gathering
- **Challenge**: agents should choose local inputs that are globally optimal



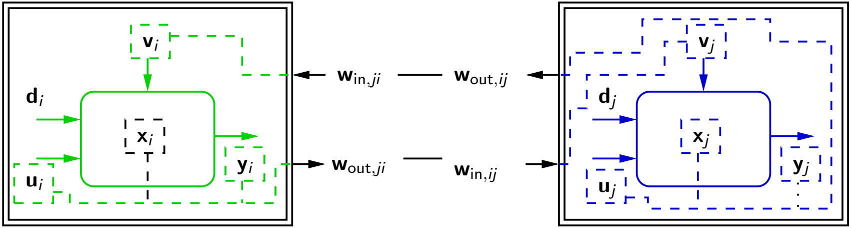


Interconnection between control agents



$$\mathbf{x}_i(k+1) = \mathbf{f}_i(\mathbf{x}_i(k), \mathbf{u}_i(k), \mathbf{d}_i(k), \mathbf{v}_i(k))$$

Interconnection between control agents



$$\mathbf{x}_i(k+1) = \mathbf{f}_i(\mathbf{x}_i(k), \mathbf{u}_i(k), \mathbf{d}_i(k), w_{\mathrm{in},j_1 i}(k), \ldots, w_{\mathrm{in},j_{m_i} i}(k))$$

$$\mathbf{w}_{\mathrm{out},ji}(k+1) = \mathbf{h}^{ji}_{\mathrm{out}}(\mathbf{u}_i(k), \mathbf{y}_i(k), \mathbf{x}_i(k+1)) \quad \text{for each neighbor } j \text{ of } i$$

Local MPC control problem of agent i at decision step k

$$\min_{\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{x}}_i(k+1)} J_{\text{local},i}(\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{x}}_i(k+1))$$

subject to

- subsystem dynamics: prediction model

$$\mathbf{x}_i(k+1) = \mathbf{f}_i(\mathbf{x}_i(k),\mathbf{u}_i(k),\mathbf{d}_i(k),\ldots)$$

$$\vdots$$

$$\mathbf{x}_i(k+N) = \mathbf{f}_i(\mathbf{x}_i(k+N-1),\mathbf{u}_i(k+N-1),\mathbf{d}_i(k+N-1),\ldots)$$

- initial local state, disturbances, and additional constraints

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$$\min_{\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{x}}_i(k+1)} J_{\text{local},i}(\tilde{\mathbf{u}}_i(k),\tilde{\mathbf{x}}_i(k+1))$$

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$$\mathbf{x}_i(k+1) = \mathbf{f}_i(\mathbf{x}_i(k),\mathbf{u}_i(k),\mathbf{d}_i(k),\mathbf{w}_{\text{in},j_1 i}(k),\ldots,\mathbf{w}_{\text{in},j_{m_i} i}(k))$$

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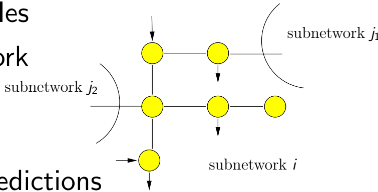
$$\mathbf{x}_i(k+N) = \mathbf{f}_i(\mathbf{x}_i(k+N-1),\mathbf{u}_i(k+N-1),\mathbf{d}_i(k+N-1),$$
$$\mathbf{w}_{\text{in},j_1 i}(k+N-1),\ldots,\mathbf{w}_{\text{in},j_{m_i} i}(k+N-1))$$

$$\mathbf{w}_{\text{out},ji}(k+N) = \mathbf{h}_{\text{out},ji}(\mathbf{u}_i(k+N-1),\mathbf{y}_i(k+N-1),\mathbf{x}_i(k+N))$$

- initial local state, disturbances and additional constraints

Interconnecting constraints

- Constraints on interconnecting variables
- Imposed by dynamics of overall network
- *What goes in into i equals what goes out from j*
- Satisfaction necessary for accurate predictions



$$
\begin{aligned}
\mathbf{w}_{\mathrm{in},ji}(k) &= \mathbf{w}_{\mathrm{out},ij}(k)\\
\mathbf{w}_{\mathrm{out},ji}(k) &= \mathbf{w}_{\mathrm{in},ij}(k)\\
\vdots \quad & \quad \vdots\\
\mathbf{w}_{\mathrm{in},ji}(k+N-1) &= \mathbf{w}_{\mathrm{out},ij}(k+N-1)\\
\mathbf{w}_{\mathrm{out},ji}(k+N-1) &= \mathbf{w}_{\mathrm{in},ij}(k+N-1)
\end{aligned}
$$

For agent controlling subsystem i

- $\mathbf{w}_{\mathrm{in},ij}$ and $\mathbf{w}_{\mathrm{out},ij}$ of neighbor j unknown
- How to make accurate predictions?
  $\rightarrow$ via negotiations

Multiple-iterations scheme to agree on values of interconnecting variables

- Each agent
  - computes optimal local *and* interconnecting variables
  - communicates interconnecting variables to neighbors
  - updates parameters $\tilde{\lambda}_{\text{in}}^{ji}, \tilde{\lambda}_{\text{out}}^{ji}$ of additional cost term J_{inter}^{i}
- Iterations continue until stopping criterion satisfied
- Scheme converges to overall optimal solution under convexity assumptions

$$\min_{\tilde{\mathbf{u}}_i, \tilde{\mathbf{x}}_i, \tilde{\mathbf{w}}_{\text{in},li}, \tilde{\mathbf{w}}_{\text{out},li}} J_{\text{local},i}(\tilde{\mathbf{u}}_i(k), \tilde{\mathbf{x}}_i(k+1)) + \sum_{j \in \text{neighbors}_i} J_{\text{inter},i}(\tilde{\mathbf{w}}_{\text{in},ji}(k), \tilde{\mathbf{w}}_{\text{out},ji}(k))$$

subject to

- dynamics of subsystem i over the horizon
- initial local state, disturbances, additional constraints

- Scheme based on augmented Lagrangian and block coordinate descent + serial implementation
- Additional objective function $J^{(s)}_{\text{inter},i}(\tilde{\mathbf{w}}_{\text{in},ji}(k), \tilde{\mathbf{w}}_{\text{out},ji}(k)) =$

$$\begin{bmatrix} \tilde{\boldsymbol{\lambda}}^{(s)}_{\text{in},ji}(k) \\ -\tilde{\boldsymbol{\lambda}}^{(s)}_{\text{out},ij}(k) \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \tilde{\mathbf{w}}_{\text{in},ji}(k) \\ \tilde{\mathbf{w}}_{\text{out},ji}(k) \end{bmatrix} + \frac{\gamma}{2} \left\| \begin{bmatrix} \tilde{\mathbf{w}}_{\text{in,prev},ij}(k) - \tilde{\mathbf{w}}_{\text{out},ji}(k) \\ \tilde{\mathbf{w}}_{\text{out,prev},ij}(k) - \tilde{\mathbf{w}}_{\text{in},ji}(k) \end{bmatrix} \right\|_2^2$$

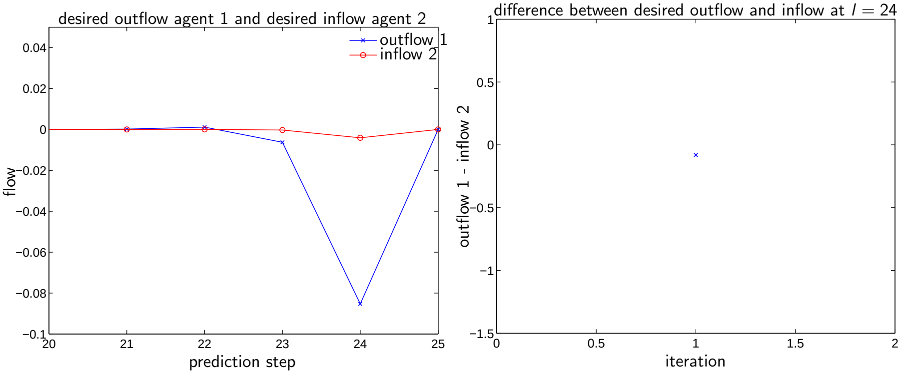
where for each j that is a neighbor that solved its problem before i in iteration s:

$$\tilde{\mathbf{w}}_{\text{in,prev},ij}(k) = \tilde{\mathbf{w}}^{(s)}_{\text{in},ij} \quad \text{and} \quad \tilde{\mathbf{w}}_{\text{out,prev},ij}(k) = \tilde{\mathbf{w}}^{(s)}_{\text{out},ij},$$

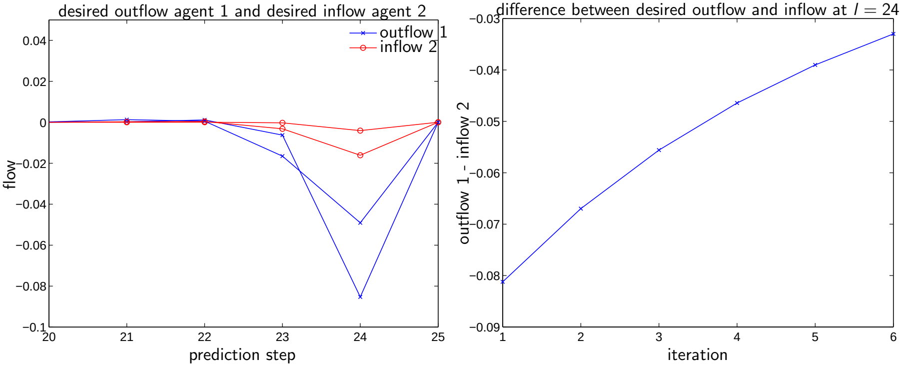
and where for each j that has not solved its problem in iteration s yet:

$$\tilde{\mathbf{w}}_{\text{in,prev},ij}(k) = \tilde{\mathbf{w}}^{(s-1)}_{\text{in},ij} \quad \text{and} \quad \tilde{\mathbf{w}}_{\text{out,prev},ij}(k) = \tilde{\mathbf{w}}^{(s-1)}_{\text{out},ij}$$

- Update of $\tilde{\boldsymbol{\lambda}}_{\text{in},ji}$:
$\tilde{\boldsymbol{\lambda}}^{(s+1)}_{\text{in},ji}(k) = \tilde{\boldsymbol{\lambda}}^{(s)}_{\text{in},ji} + \gamma \left(\tilde{\mathbf{w}}^{(s)}_{\text{in},ji}(k) - \tilde{\mathbf{w}}^{(s)}_{\text{out},ij}(k) \right)$



Obtaining agreement on flows between two subsystems



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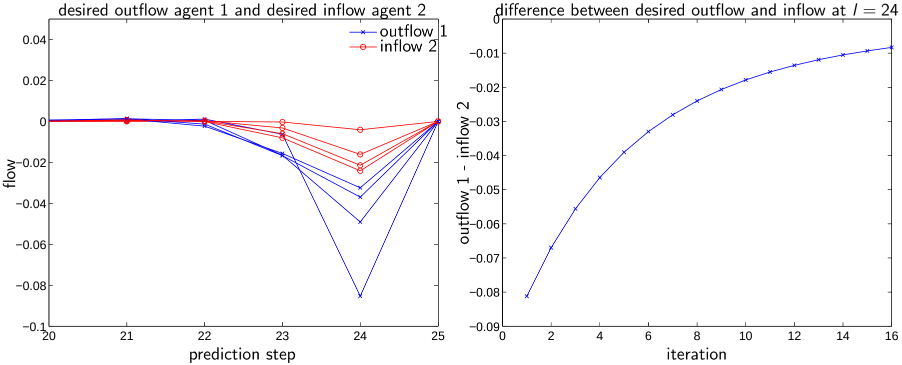

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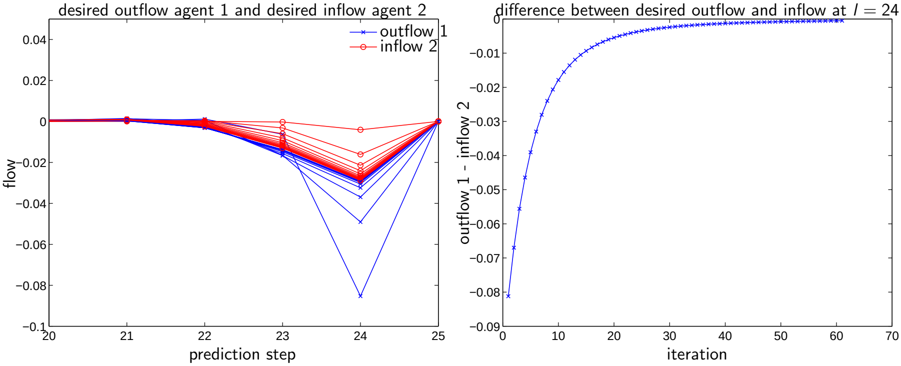
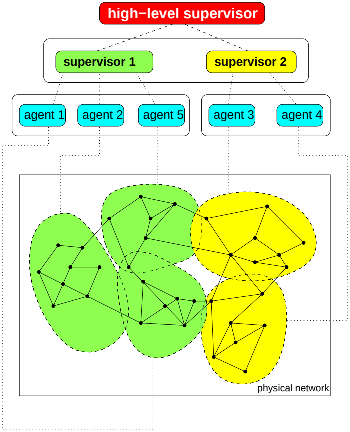
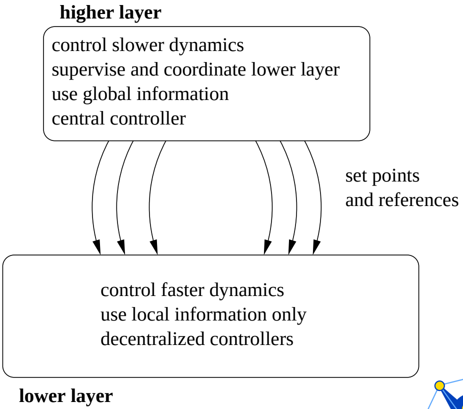


Obtaining agreement on flows between two subsystems

- Multi-level control with intelligent control agents & coordination
- Time-based and space-based separation into layers

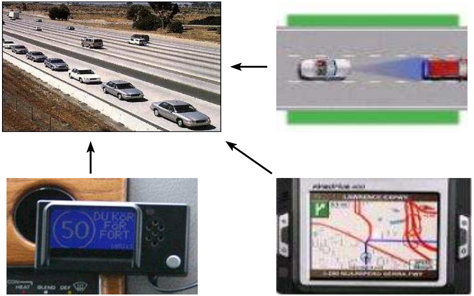




Example: Intelligent Vehicle Highway Systems (IVHS)

- Integrate various in-vehicle and roadside-based traffic control measures that support platoons of fully autonomous intelligent vehicles

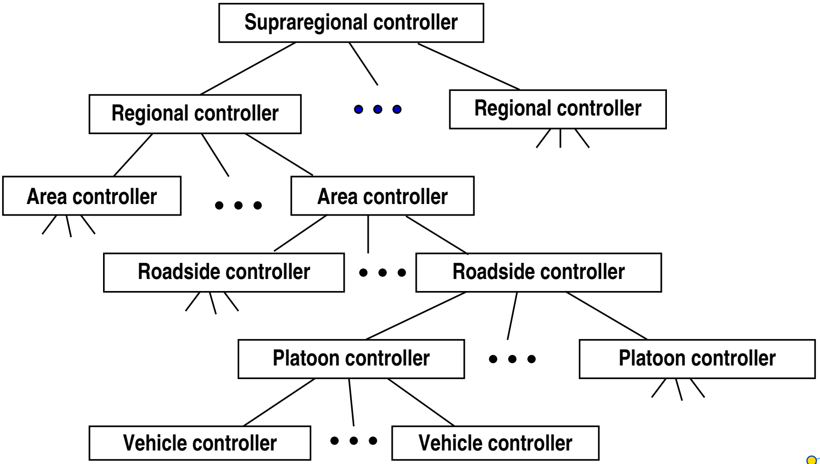
platoon cooperative adaptive cruise control



intelligent speed adaptation dynamic route guidance

- Goal: improved traffic performance (safety, throughput, environment, . . .) + constraints (robustness, reliability, . . .)

HD-MPC approach for IVHS (∼ California PATH)



Controller	Unit	Control	Time scale
Vehicle	vehicle	throttle, brake, steering	$\ll$ s
Platoon	vehicles	distances & speeds, trajectories	$<$ s
Roadside	platoons	lanes & speeds, split & merge	s–min
Area	flows of platoons	routing	$>$ min
Regional	flows	routing	$>$ 15–30 min

Control strategies

- Vehicle controllers: (adaptive) PID + logic (for safety)
- Platoon controllers: rule-based control, hybrid control
- Roadside, area, regional controllers: MPC

$$\min_{u(k),\ldots,u(k+N_c-1)} J(k)$$
$$\text{s.t. model of system}$$
$$\text{operational constraints}$$

→ medium-sized problems due to temporal & spatial division
→ still tractable

- Coordination (top-down) via performance criterion J or constraints

Roadside controllers

- Control highway or stretch of highway
- Measurements: position, speed, lanes of platoon leaders
- Control inputs: platoon speeds, lane allocations, on-ramp release times
- Objectives:
  - track speed and splitting rate profiles imposed by area controllers
  - minimize total time spent (TTS) in network and queues, . . .
- Constraints: min. headway, min. and max. speeds

MPC for roadside controllers

- Model: "big-car" model
  platoon = vehicle with speed-dependent length

$$L_{\text{platoon},p}(k) = (n_p - 1)S_0 + \sum_{i=1}^{n_p-1} T_{\text{head},i} v_{n_p}(k) + \sum_{i=1}^{n_p} L_i$$

  with S_0 minimum safe distance at zero speed and $T_{\text{head},i}$ the desired time headway

- Nonlinear optimization problem:

$$\begin{aligned} &\min \left(\text{TTS links} + \text{TTS queues}\right) \\ &\text{subject to nonlinear model} \\ &\qquad\qquad\quad \text{operational constraints} \end{aligned}$$

- Optimization: mixed-integer nonlinear programming
  Simplify by bi-level approach in which first lane allocation is determined (heuristics, optimized, slower rate, ...)

Area controllers

- Route guidance + provide set-points for roadside controllers
- Traffic network is represented by graph with nodes and links
- Due to computational complexity, optimal route choice control done via flows on links
- Optimal route guidance: nonlinear integer optimization with high computational requirements $\rightarrow$ intractable

Area controllers

- Fast approaches based on
  - Mixed-Integer Linear Programming (MILP)
    - model describes flows and queues
    - transform nonlinear problem into system of linear equations using binary variables
    - can be solved efficiently using branch-and-bound; several efficient commercial and freeware solvers available
  - macroscopic traffic flow model
    - model describes flows, densities, speeds, and queues
      model is based on model for human drivers (METANET)
    - for humans, splitting rates are determined by traffic assignment
    - in IVHS, splitting rates considered as controllable input

Regional controllers

- Control collection of areas
- Determine optimal flows of platoons between areas
- Model: aggregate model – based on IVHS variant of static density-flow relation
- Optimization: Nonlinear non-convex programming problem
  Will be approximated using mixed-integer linear programming

- How to obtain tractable prediction models?
- What is the best division into subsystems?
- Selection of static/dynamic region boundaries?
- How to determine subgoals so as to optimize overall goal?
- How should the higher-level control layers be designed?
- How to effectuate interaction and coordination between agents and control regions?
- How to resolve conflicts & prevent counteracting?
- How can existing approaches be extended to hybrid systems?
- How can the computation/iteration time be reduced? (algorithms, properties, approximations, reductions, . . .)
- Analysis (stability, reliability, robustness, . . .)