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Hierarchical MPC with applications in transportation and infrastructure networks

Milan, Italy, August 28, 2011

HD–MPC

SEVENTH FRAMEWORK PROGRAMME

1. HD-MPC for large-scale systems
2. Traffic management and automated highway systems
3. Multi-level multi-scale HD-MPC for AHS
4. Related work
5. Conclusions and future work

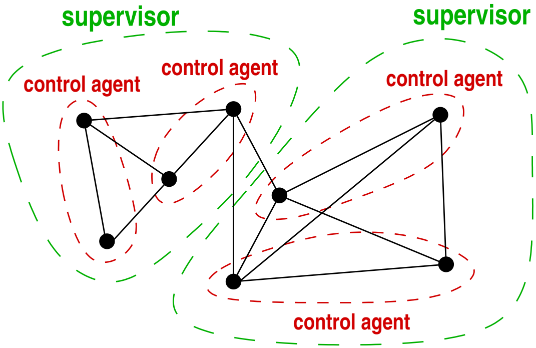
Challenges in control of large-scale networks:

- Large-scale networks
- Distributed vs centralized control
- Optimality $\leftrightarrow$ computational efficiency/tractability
- Global $\leftrightarrow$ local
- Scalability
- Communication requirements (bandwidth)
- Robustness against failures

$\longrightarrow$ multi-level multi-agent approach

Multi-level multi-agent control

- Multi-level control with intelligent control agents & coordination
- Time-based and space-based separation into layers



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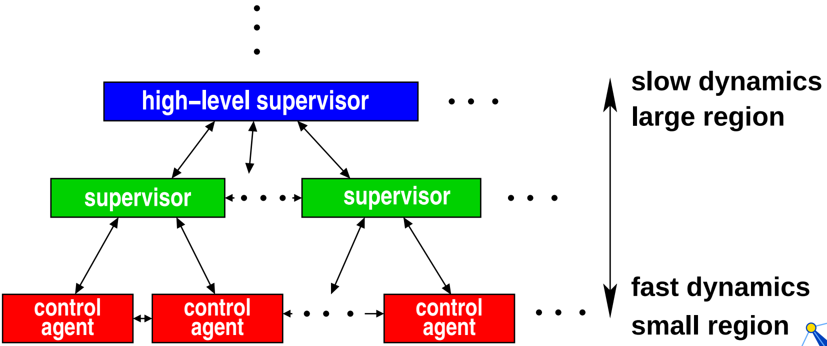


Multi-level control framework

- Lowest level:
  - local control agents
  - "fast" control
  - small region
  - operational control
- Higher levels:
  - supervisors
  - "slower" control
  - larger regions
  - operational, tactical, strategic control
- Multi-level, multi-objective control structure
- *Coordination at and across all levels*
- Combine with model predictive control (MPC)

Major problem for MPC in practice: Required computation time for large-scale systems

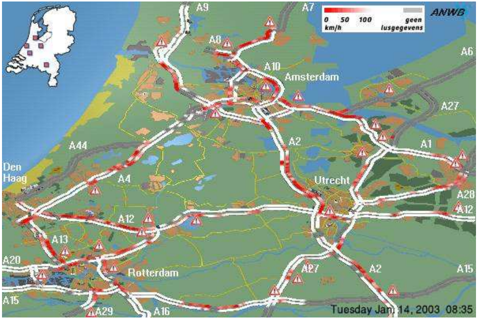
- Use distributed and/or hierarchical control approach
- Choice of the prediction model: accuracy versus computational complexity
- Right optimization approach
  - parallel and/or distributed optimization
  - approximate original MPC optimization problem by another optimization problem that can be solved efficiently
- Include application-specific knowledge

Need for traffic control

Traffic jams & congestion

$\longrightarrow$ cause time losses, extra costs, more incidents

have negative impact on economy, environment, society



Several ways to reduce traffic jams and to improve traffic performance:

- New infrastructure, missing links
- Pricing
- Modal shift
- Better use of available capacity through intelligent traffic control

Intelligent traffic control

Next generation traffic control and management system

- Use in-car telematics (navigation, telecommunication, information, . . .) systems
- Vehicle-vehicle + vehicle-roadside communication
- Use intelligent vehicles (IVs)
  - control system senses environment using sensors
  - enhances either performance of driver or vehicle itself
    - assisting (advisory/warning)
    - taking partial or complete control (full automation)
- Two variants of traffic management using IVs:
  - cooperative vehicle-infrastructure systems (CVIS): drivers are still in charge of their vehicles
  - Automated Highway Systems (AHS): autonomous vehicles organized in platoons

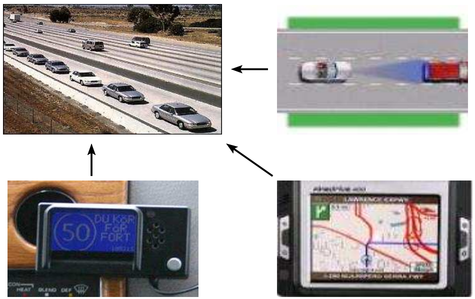
Automated highway systems (AHS)

- Platoons of intelligent, autonomous vehicles
- Small inter-vehicle distance inside distances + high speeds
  $\rightarrow$ higher throughput
- Larger inter-platoon distance for safety
- Problems:
  - transition
  - psychological & legal aspects

  $\rightarrow$ long-term, trucks

Automated highway systems (AHS)

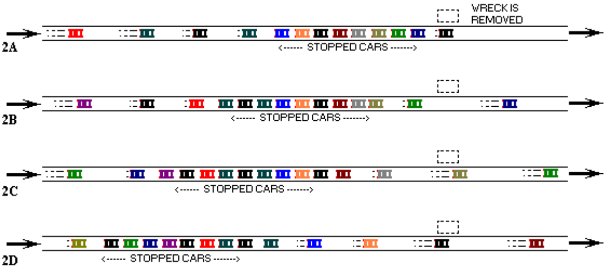
- Integrate various in-vehicle and roadside-based traffic control measures that support platoons of fully autonomous IVs



- Goal: improved traffic performance (safety, throughput, environment, ...) + constraints (robustness, reliability, ...)

Additional advantage of platoons: No capacity drop

- Capacity drop for human drivers: If traffic flow breaks down, then afterwards outflow from congested area is less than previous higher flow



- Reason: Human drivers tend to accelerate more slowly when they are coming out of congestion
- This effect plays less or even not with autonomous vehicles

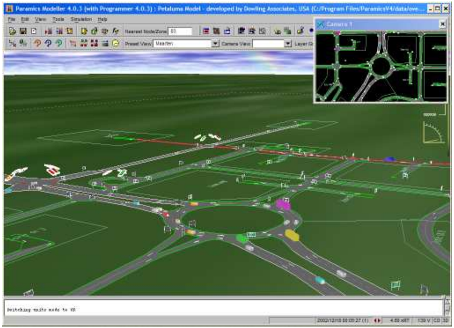
Traffic flow models

Two main classes:

- Microscopic models → individual vehicles
- Macroscopic models → aggregated variables

Microscopic traffic flow models

- Consider individual vehicles
- Car following + lane changing + overtaking models
- Different driver classes (with different parameters settings)
- Simulation rather time-consuming for large networks
  $\longrightarrow$ less suited as prediction model for MPC
  $\longrightarrow$ better suited as simulation/validation model



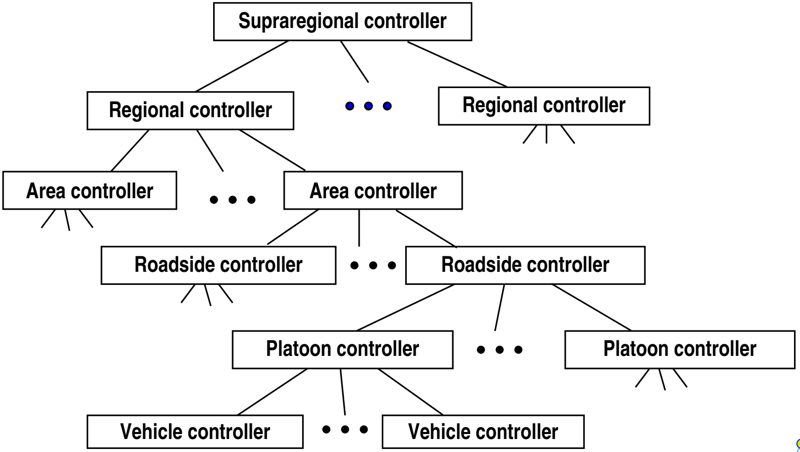
Macroscopic traffic flow models

- Work with aggregated variables (average speed, density, flow)
- Examples:
  - fluid-like models: Lighthill-Whitham-Richards (LWR), Payne, METANET, ...
  - gas-kinetic models: Helbing model, ...
- Trade-off between computational speed versus accuracy

  → well suited as prediction model for MPC

  → less suited as simulation/validation model
- In this presentation we use macroscopic models *for automated highway systems* as prediction model for MPC

A multi-level multi-scale HD-MPC approach for AHS →
hierarchical multi-layer control approach (∼ California PATH)



Controller	Unit	Control	Time scale
Vehicle	vehicle	throttle, brake, steering	$\ll$ s
Platoon	vehicles	distances & speeds, trajectories	$<$ s
Roadside	platoons	lanes & speeds, split & merge	s–min
Area	flows of platoons	routing	$>$ min
Regional	flows	routing	$>$ 15–30 min

Control strategies

- Vehicle controllers: (adaptive) PID + logic (for safety)
- Platoon controllers: rule-based control, hybrid control
- Roadside, area, regional controllers: MPC

$$\begin{aligned} \min_{u(k),\ldots,u(k+N_c-1)} \; & J(k) \\ \text{s.t.} \; & \text{model of system} \\ & \text{operational constraints} \end{aligned}$$

→ medium-sized problems due to temporal & spatial division
→ still tractable

- Coordination (top-down) via performance criterion J or constraints

Roadside controllers

- Control highway or stretch of highway
- Measurements: position, speed, lanes of platoon leaders
- Control inputs: platoon speeds, lane allocations, on-ramp release times
- Objectives:
  - track speed and splitting rate profiles imposed by area controllers
  - minimize total time spent (TTS) in network and queues, ...
- Constraints: min. headway, min. and max. speeds

MPC for roadside controllers

- Model: "big-car" model
  platoon = vehicle with speed-dependent length

$$L_{\text{platoon},p}(k) = (n_p - 1)S_0 + \sum_{i=1}^{n_p-1} T_{\text{gap},i} v_{n_p}(k) + \sum_{i=1}^{n_p} L_i$$

  with S_0 minimum safe distance at zero speed and $T_{\text{gap},i}$ the desired time gap

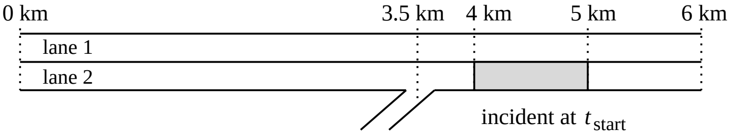
- Nonlinear optimization problem:

$$\begin{aligned} &\min \big(\text{TTS links} + \text{TTS queues}\big) \\ &\text{subject to } \text{nonlinear model} \\ &\qquad\qquad\quad\ \text{operational constraints} \end{aligned}$$

- Optimization: mixed-integer nonlinear programming
  Simplify by bi-level approach in which first lane allocation is determined (via heuristics, optimized, slower rate, . . .)

Case study – Problem statement

Two-lane highway with an incident causing traffic



Scenario:

- Demand: 2500 veh/h (mainstream) and 350 veh/h (on-ramp)
- Incident at 4-5 km, start of simulation (10 minutes)
- Queues at start: empty
- Simulation period: 10 min, controller sampling time: 1 min
- Simulation sampling time: 1 s

Case study – Cases

Cases considered:

- Uncontrolled human drivers
- Controlled human drivers (current situation)
- Platoon approach – our approach

Case study – Results

Case	TTS (veh·h)	Relative improvement (%)
Uncontrolled	71.80	0 %
Controlled (human drivers)	63.38	10.96 %
Controlled (platoons)	57.75	18.86 %

Reduced TTS $\longrightarrow$ decreased travel times, increased trips, . . .

Area controllers

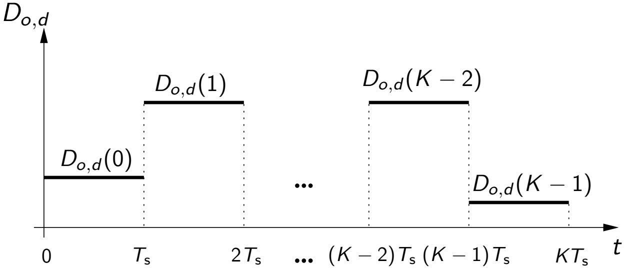
- Route guidance + provide set-points for roadside controllers
- Traffic network is represented by graph with nodes and links
- Due to computational complexity, optimal route choice control done via flows on links
- Optimal route guidance: nonlinear *integer* optimization with high computational requirements → intractable

Area controllers (contd.)

- Fast approaches based on
  - Mixed-Integer *Linear* Programming (MILP)
    - transform nonlinear problem into system of linear equations using binary variables
    - can be solved efficiently using branch-and-bound; several efficient commercial and freeware solvers available
  - macroscopic METANET-like traffic flow model
    - for humans, splitting rates are determined by traffic assignment
    - in AHS, splitting rates considered as controllable input
    - will result in non-convex *real-valued* optimization

MILP approach – General set-up

- Only consider flows and queue lengths
- Each link has maximal allowed capacity constraint
- Piecewise constant time-varying demand - $[kT_s, (k+1)T_s)$ for $k = 0, \ldots, K-1$ with K (simulation horizon)



- Main goal: assign optimal flows $x_{l,o,d}(k)$

MILP approach – Model

- Inflow at origin:

$$\sum_{l \in L_o^{\text{out}} \cap L_{o,d}} x_{l,o,d}(k) \leqslant D_{o,d}(k) + \frac{q_{o,d}(k)}{T_\text{s}} \quad \text{for each } d \in \mathcal{D}$$

- Outflow from origin to destination:

$$F_{o,d}^{\text{out}}(k) = \sum_{l \in L_o^{\text{out}} \cap L_{o,d}} x_{l,o,d}(k)$$

- Assume constant delay κ between beginning and end of link
- Queue behavior at origin: Total demand − outflow
- More specifically, $D_{o,d}(k) - F_{o,d}^{\text{out}}(k)$ in time interval $[kT_\text{s}, (k+1)T_\text{s})$

$$q_{o,d}(k+1) = \max\left(0,\ q_{o,d}(k) + \left(D_{o,d}(k) - F_{o,d}^{\text{out}}(k)\right)T_\text{s}\right)$$

MILP approach – Equivalences

P1: $[f(x) \leqslant 0] \iff [\delta = 1]$ is true if and only if

$$\begin{cases} f(x) \leqslant M(1-\delta) \\ f(x) \geqslant \epsilon + (m-\epsilon)\delta \end{cases}$$

P2: $y = \delta f(x)$ is equivalent to $\begin{cases} y \leqslant M\delta \\ y \geqslant m\delta \\ y \leqslant f(x) - m(1-\delta) \\ y \geqslant f(x) - M(1-\delta) \end{cases}$

- f function with upper and lower bounds M and m
- δ is a binary variable
- y is a real-valued scalar variable
- ϵ is a small tolerance (machine precision)

$\longrightarrow$ transform max equations into MILP equations

MILP approach – Transforming the queue model

$$q_{o,d}(k+1) = \max\left(0,\ q_{o,d}(k) + (D_{o,d}(k) - F_{o,d}^{\text{out}}(k))\,T_{\text{s}}\right)$$

Define

$$[\ \delta_{o,d}(k) = 1\ ] \iff [\ q_{o,d}(k) + (D_{o,d}(k) - F_{o,d}^{\text{out}}(k))\,T_{\text{s}} \geqslant 0\ ]$$

Can be transformed into MILP equations using equivalence P1

$$q_{o,d}(k+1) = \delta_{o,d}(k)\Big(\underbrace{q_{o,d}(k) + (D_{o,d}(k) - F_{o,d}^{\text{out}}(k))\,T_{\text{s}}}_{f\ \text{(linear)}}\Big)$$

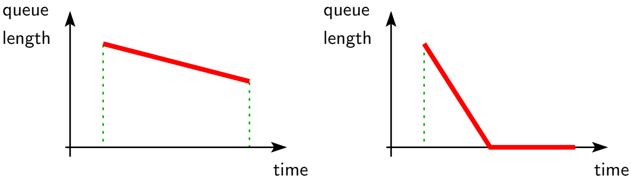
$$= z_{o,d}(k)$$

Product between $\delta_{o,d}(k)$ and f can be transformed into system of MILP equations using equivalence P2

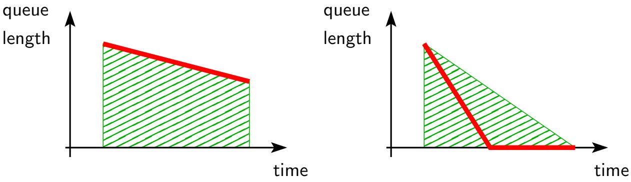
Queue model $\longrightarrow$ system of MILP equations

MILP approach – Objective function for queues

Original objective function: time spent in queues (linear/quadratic):



Approximated objective function (linear):



MILP approach – Objective Functions

- Time spent in links:

$$J_{\text{links}} = \sum_{k=0}^{K_{\text{end}}-1} \sum_{(o,d)\in\mathcal{O}\times\mathcal{D}} \sum_{l\in L_{o,d}} x_{l,o,d}(k)\kappa_l T_{\text{s}}^2$$

- Time spent in queues:

$$J_{\text{queue}} = \sum_{k=0}^{K_{\text{end}}-1} \sum_{(o,d)\in\mathcal{O}\times\mathcal{D}} \frac{1}{2}(q_{o,d}(k) + q_{o,d}(k+1))T_{\text{s}}$$

MILP approach – Overall area control problem

Nonlinear optimization problem:

$$\min \left(\text{TTS links} + \text{TTS queues}\right)$$

subject to

nonlinear model

operational constraints

MILP optimization problem:

$$\min \left(\text{TTS links} + \widehat{\text{TTS}} \text{ queues}\right)$$

subject to

MILP model

operational constraints

MILP approach – Case study

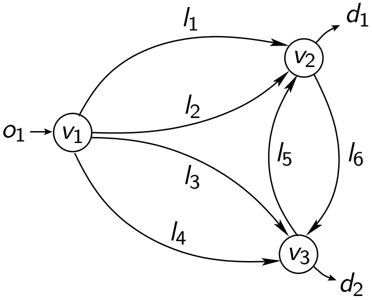


Figure: Set-up of case study network

MILP approach – Case study – Set-up

- Dynamic demand case with queues only at origins of network

Period (min)	0–10	10–30	30–40	40–60
D_{o_1,d_1} (veh/h)	5000	8000	2500	0
D_{o_1,d_2} (veh/h)	1000	2000	1000	0

- Scenario:
  - simulation period: 60 min, sampling time: 1 min
  - capacities: C_1=1900 veh/h, C_2=2000 veh/h, C_3=1800 veh/h, C_4=1600 veh/h, C_5=1000 veh/h, and C_6=1000 veh/h
  - delay factor: κ_1=10, κ_2=9, κ_3=6, κ_4=7, κ_5=2, and κ_6=2

MILP approach – Case study – Cases

Cases considered

- Case A: no control
- Case B: controlled using the MILP solution
- Case C: controlled using the exact solution

MILP approach – Case study – Results

Case	TTS_{tot} (veh.h)	improvement	CPU time (s)
No control	1434	0 %	–
MILP	1081	24.6 %	0.27
SQP (5 initial points)	1067	25.6 %	90.0
SQP (50 initial points)	1064	25.8 %	983
SQP (with MILP solution as initial point)	1064	25.8 %	1.29

MILP approach – Case study – Analysis

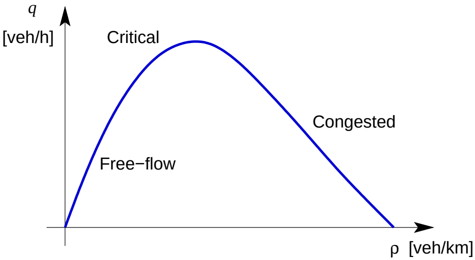
- Uncontrolled case: only direct/short routes are used. Length of origin queue increases with time
- Controlled cases: flows assigned to both short and long routes

Regional controllers

- Control collection of areas
- Determine optimal flows of platoons between areas
- Model: aggregate model – AHS variant of the Macroscopic Fundamental Diagram (MFD)
- Optimization: Nonlinear non-convex programming problem
  Will be approximated using mixed-integer linear programming

Macroscopic Fundamental Diagram (MFD)

- Introduced by Geroliminis and Daganzo
- Describes relation between space-mean flow and density in neighborhood-sized sections of cities (up to 10 km^2)
- Macroscopic fundamental diagram is independent of the demand
- Outflow of area is proportional to space-mean flow within area



Macroscopic Fundamental Diagram for AHS

- Adopt modified version of MFD for AHS
- Shape of MFD will be sharper and maximal flow will be much higher than in MFD for human drivers
- Represent AHS network by graph
  - links correspond to areas, with inflow $q_{\text{in},a}(k)$, outflow $q_{\text{out},a}(k)$, and density $\rho_a(k)$
  - nodes correspond to connections between areas, external origins (with inflow $q_{\text{orig},o}(k)$), or external exits (with outflow $q_{\text{exit},e}(k)$)

Model for regional controllers

- Network MFD for AHS results in static description of form

$$q_{\text{out},a}(k) = \mathcal{M}_a(\rho_a(k))$$

- Evolution of densities inside each area is described using simple conservation equation:

$$\rho_a(k+1) = \rho_a(k) + \frac{T}{L_a}(q_{\text{in},a}(k) - q_{\text{out},a}(k))$$

  with T sample time step system and L_a measure for total length of highways and roads in area a

- For every node ν balance between inflows and outflows:

$$\sum_{a\in\mathcal{I}_\nu} q_{\text{out},a}(k) + \sum_{o\in\mathcal{I}_{\text{orig},\nu}} q_{\text{orig},o}(k) = \sum_{a\in\mathcal{O}_\nu} q_{\text{in},a}(k) + \sum_{e\in\mathcal{O}_{\text{exit},\nu}} q_{\text{exit},e}(k)$$

MPC for regional controllers

- Try to keep density in each region below critical density $\rho_{\mathrm{crit},a}$:

$$J_{\mathrm{pen}}(k) = \sum_{j=1}^{N_{\mathrm{p}}} \sum_{a} \left[\max(0, \rho_a(k+j) - \rho_{\mathrm{crit},a}) \right]^2$$

- Also minimize total time spent (TTS) by all vehicles in region:

$$J_{\mathrm{TTS}}(k) = \sum_{j=1}^{N_{\mathrm{p}}} \sum_{a} L_a \rho_a(k+j) T$$

- Total objective function:

$$J(k) = J_{\mathrm{pen}}(k) + \gamma J_{\mathrm{TTS}}(k)$$

- Constraints on maximal flows from one area to another,. . .
- Results in nonlinear, non-convex optimization problem

Mixed integer linear programming (MILP) – Two properties

- Given function f with lower bound m and upper bound M
- **Property 1**:
  $[f(x) \leq 0] \Leftrightarrow [\delta = 1]$ is equivalent to

$$\begin{cases} f(x) \leq M(1-\delta) \\ f(x) \geq \varepsilon + (m-\varepsilon)\delta \end{cases}$$

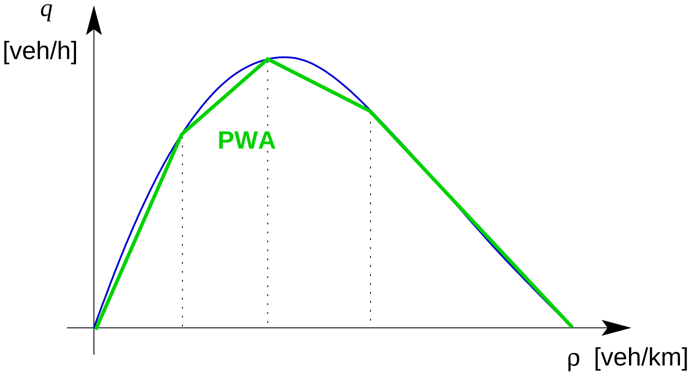
- **Property 2**:
  $y = \delta f(x)$ with $\delta \in \{0, 1\}$ is equivalent to

$$\begin{cases} y \leq M\delta \\ y \geq m\delta \\ y \leq f(x) - m(1-\delta) \\ y \geq f(x) - M(1-\delta) \end{cases}$$

Transformation into MILP problem

- Approximate MFD by Piece-Wise Affine (PWA) function

$$q_{\mathrm{out},a}(k) = \alpha_{a,i}\rho_a(k) + \beta_{a,i} \quad \text{if } \rho_a(k) \in [\rho_{a,i}, \rho_{a,i+1}]$$



Transformation into MILP problem

- Approximate MFD by Piece-Wise Affine (PWA) function

$$q_{\text{out},a}(k) = \alpha_{a,i}\rho_a(k) + \beta_{a,i} \ \text{ if } \rho_a(k) \in [\rho_{a,i}, \rho_{a,i+1}]$$

- Introduce binary variables $\delta_{a,i}(k)$ such that

$$\delta_{a,i}(k) = 1 \ \text{ if and only if } \ \rho_{a,i} \leq \rho_a(k) \leq \rho_{a,i+1}$$

  Can be transformed into MILP equations using Property 1
- Now we have

$$q_{\text{out},a}(k) = \sum_{i=1}^{N_a} (\alpha_{a,i}\rho_a(k) + \beta_{a,i})\delta_{a,i}(k)$$

- Introduce real-valued auxiliary variables $y_{a,i}(k) = \rho_a(k)\delta_{a,i}(k)$
  Can be transformed into MILP equations using Property 2

Transformation into MILP problem

- Results in

$$q_{\mathrm{out},a}(k) = \sum_{i=1}^{N_a} \alpha_{a,i} y_{a,i}(k) + \beta_{a,i}\delta_{a,i}(k)$$

- If we combine all equations and inequalities, we obtain a system of mixed-integer linear inequalities

Transformation into MILP problem

- Recall

$$J_{\text{pen}}(k) = \sum_j \sum_a \left[\max(0, \rho_a(k+j) - \rho_{\text{crit},a})\right]^2 \quad \rightarrow \text{not linear}$$

$$J_{\text{TTS}}(k) = \sum_j \sum_a L_a \rho_a(k+j) T \quad \rightarrow \text{linear!}$$

- Removing square in $J_{\text{pen}}(k)$ results in PWA objective function
  Can be transformed in MILP equations using Properties 1 & 2
- Hence, we get MILP problem
- Solution of MILP problem can be directly applied or it can be used as good initial starting point for original nonlinear, non-convex MPC optimization problem

Related work: Traffic management using MPC

- More viable option on short term:
  roadside intelligence
  → traffic control center +
  current infrastructure
- Use conventional control measures:
  variable speed limits, ramp metering,
  traffic signals, lane closures, shoulder lane
  openings, tidal flow, . . .
- Also include “soft” control measures:
  dynamic route information, travel time
  information, . . .

Ongoing research

- Address complexity issues for large-scale systems
  - simplified models for urban traffic networks
  - parametrized MPC
- Alternative objective functions + related models
  - emissions: CO, NO_x, CO_2, HC, ...
  - fuel consumption

Cooperative Vehicle Infrastructure Systems

- Intermediate step between current system and AHS

Other applications

- Electricity networks
- Water networks
- Railway networks
- Logistic systems




Conclusions

- Hierarchical control framework for automated highway systems (AHS)
- Focus on roadside, area, and regional controllers
- In general: nonlinear, non-convex mixed-integer optimization problems
- Reduce complexity of problem by selecting appropriate models and making approximations
- Results by bi-level, mixed-integer linear programming, or nonlinear, non-convex real-valued optimization problems

Future work

- extensive integrated case study & assessment
- further development of HD-MPC approaches
- further improvements in efficiency and performance

Main issues and topics in HD-MPC for transportation and infrastructure networks

- How to obtain tractable prediction models?
- What is the best division into subnetworks?
- Selection of static/dynamic region boundaries?
- How to determine subgoals so as to optimize overall goal?
- How can existing approaches be extended to hybrid systems?
- How can the computation/iteration time be reduced further? (algorithms, properties, approximations, reductions, ...)