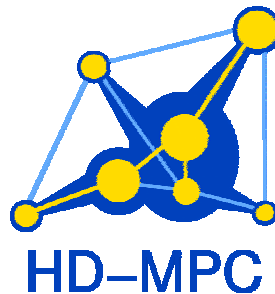


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Table of Content

1	INTRODUCTION	4
2	HPV DESCRIPTION (PROCESS AND CONTROL).....	5
2.1	PROCESS DESCRIPTION	5
2.1.1	<i>The components</i>	5
2.1.2	<i>The Valley</i>	7
2.2	CONTROL DESCRIPTION	9
3	LITERATURE SURVEY ON MODELLING AND CONTROL	10
4	HD-MPC CONTROL SPECIFICATION FOR HPV	11
4.1	PROCESS DESCRIPTION	11
4.2	OBJECTIVE FUNCTIONS	12
4.3	CONSTRAINTS FUNCTIONS	13
4.3.1	<i>Local constraints</i>	13
4.3.2	<i>global constraints</i>	14
4.4	ROBUSTNESS ISSUES	14
4.4.1	<i>Perturbation uncertainties</i>	14
4.4.2	<i>Model uncertainties</i>	15
4.5	TEST SCENARIOS	15
4.5.1	<i>Nominal tests</i>	15
4.5.2	<i>Robustness tests</i>	15
4.5.3	<i>Off-design tests</i>	15
5	HIERARCHICAL DECOMPOSITION OF CONTROL.....	16
5.1	HD-MPC ARCHITECTURE	16
5.2	DEFINITIONS & ASSUMPTIONS	17
5.2.1	<i>Primal and dual problem</i>	17
5.2.2	<i>Additivity</i>	18
5.3	PRICE DECOMPOSITION	18
5.4	QUANTITY DECOMPOSITION	19
5.5	PREDICTION DECOMPOSITION	19
5.6	TEMPORAL OR CASCADE DECOMPOSITION	20
6	CONCLUSION	20

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Executive Summary

The report presents the control specifications for the control of an Hydro power Valley. For this application, the objectives, the constraints and the robustness requirements are given. This information will be used as input for the HD-MPC design.

1 Introduction

Hydro Power is an essential renewable mean of power generation that represented in 2006 16 % of the world production [1]. In some countries the part of hydro can even exceed 80 % like in Norway and Brazil and hydro power plants must be flexible enough to adjust the electrical power production to the demand. In order to reduce the CO₂ emission, the part of hydro plant will probably be bigger in the future. In the same time, climate change will probably impact the availability of water and efficient management of this resource will be at stake.

In Europe, a goal of 22% of renewable energy is aimed for 2010 to limit the CO₂ emission. Hydro is one of the main renewable energies with wind and solar energy. The hydro power plants are already well developed and for instance almost 70% of the possible sites are used in France. A qualitative improvement is still possible by using an optimal real-time management of the resources that will maximize the efficiency of the hydro power plants. This management must be compatible with the other uses of the hydro resources (irrigation, navigation, tourism), with environmental and ecological requirements and with the safety which imposes constraints on level, flow rate at given points of the hydro power valley.

A coordination of the hydropower plants of a given valley can produce a gain in efficiency and increase the power production and the maneuverability while respecting the constraints. In particular, the maneuverability of the hydropower plants is an important feature as it can be seen in Figure 1 that presents the hydro power solicitation during the electricity peak demand in early 2009.

Basically the problem of optimizing an hydro power valley (HPV) is to maximize the energy over a period while respecting operating constraints.

Another relevant goal for hydropower valley is to increase the ancillary services and particularly the frequency response.

Objectives related to safety and environmental requirements are generally formulated as domain limitation (level, flowrate, ...) for the control purpose and are not objective in the sense of the optimization problem;

MPC is thought to be a promising approach to reach these goals.

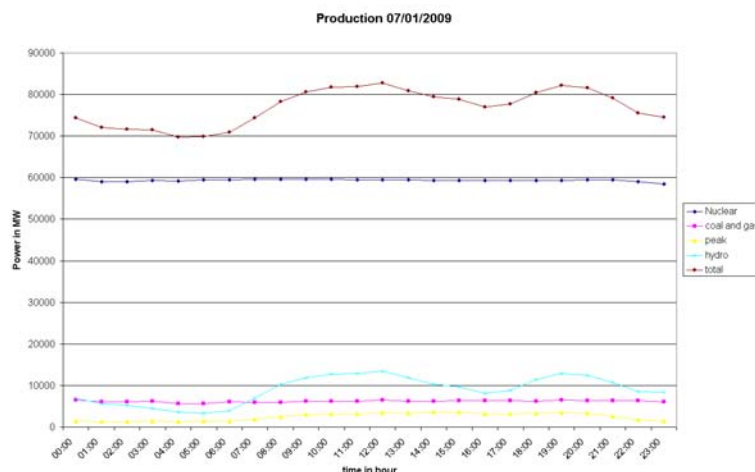


Figure 1 : hydro power solicitation during peak demand in early 2009. source www.rte-france.fr

Today, each of the plants of the HPV is controlled by decentralized power and level PID control with references determined by off-line optimization. In order to improve the efficiency of the HPV, an on-line optimal control with MPC is thought. A fully centralized solution is not envisaged in the future for several reasons.

The first reason is the complexity of the plant that will probably lead to computation requirements not achievable by the current DCS.

The second reason is that in case of loss of communication between the HPV optimizer located in the control room and the local controllers, the local controllers must be able to regulate the level and the

power at the last reference.

Another reason is that the modifications of the control architecture must be as limited as possible to facilitate the implementation.

Distributed control seems to be a good intermediary between decentralized and centralized control. Decomposition/coordination methods consist in dividing an MPC optimization problem into several sub-problems, which are to be coordinated so as to get the overall optimization.

In the HD-MPC project, we will study the application of distributed optimal control for large scale water systems to improve the efficiency and the management of an HPV. This report is the first stage of this study and gives the control specification for the searched solution.

In the first part, a HPV case study description is given with the components, the operating modes, and the operating constraints. We are interested in a HPV that globally consists of a cascade run-of-river plants and storage lakes. For the present work, a simplified version of this study case HPV with few reaches and lakes will be considered. A brief description of the existing control system and its architecture is also given. This system is controlled by a hierarchical control, with local controllers (Programmable Logic Controller) used to regulate the level and power of each plants. The upper level is dedicated to operator HMI. Our aim is to propose and develop at the higher level an algorithm to be implemented in the control center (DCS) that controls the whole valley.

The second part is a literature survey of models and advanced control solutions that have been applied to HPV and that could be useful to develop and HD-MPC solution.

The third part is the control specification for the study case that will be considered in the HD-MPC project. It presents the control objective, the constraints and robustness requirements as well validation tests.

The last part proposes several HD-MPC architectures for the HPV optimization problem.

2 HPV Description (process and control)

2.1 Process description

A HPV is a large scale system that is composed of several components linked together. In this section the different equipments are presented before a description of the valley that will be considered in the HD-MPC project.

2.1.1 The components

Dams :

Dams are elevations build on the river to increase the water level, to create a storage and to force the water to flow through diversions. Dams are equipped with spillways that control their level in case of flood. In normal operations a small part of the water is not derived and flows through the natural river. This ecological flow rate is imposed for environmental reason. Dams are also equipped with other devices to withdraw the mud, to let the fish pass and to empty the reservoir for inspection.

Locks :

When the river is used for navigation, locks are needed to let the barge pass the dams. The filling and emptying of the lock creates flow perturbations on the river, but they are neglected in this study.



Figure 2 : A Dam



Figure 3 : A Lock

Lakes :

Lakes are reservoirs that collect and store the water of the main rivers and of the small creeks on the mountains sides.

Pumps :

It can be useful to pump water from one small reservoir to a bigger one equipped with a turbine. Pumps connect the lakes together and increase the storage size. Pumping operations are in general done when the electrical power demand is low.

Turbines :

The hydro power is converted into mechanical power by the turbine in the power house. The mechanical power is then converted in electrical power by a generator connected to the grid. The turbine is fed by water coming from the reservoir through a penstock. For high head¹, a surge tank is added at the top of the penstock to avoid water hammer. The electrical power Pe^2 depends on the head H and the flow rate Qe that enters through the turbine. Several technology are available to cover a large domain of head and flow rate values : Pelton are used for higher head, Francis and Kaplan for medium and low head.

In general the power house consists of several units (turbine/generator compound) connected to the same bus bar.

A Pumped Storage Hydroelectric station is able to pump and turbine the water and is used by the company to store indirectly electrical power. The water is pumped when the demand is low and turbined when the demand is high.

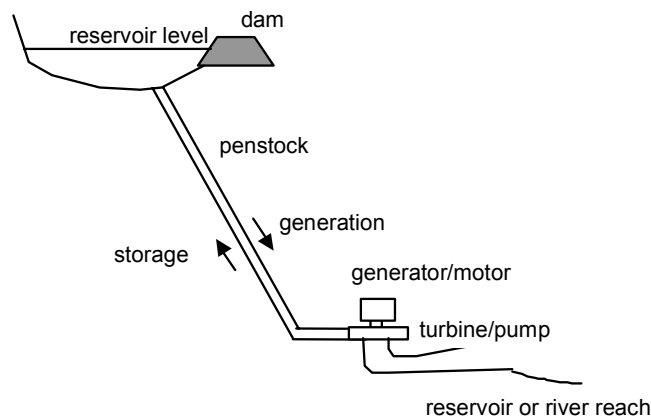


Figure 4 : Pumped Storage Hydroelectric station

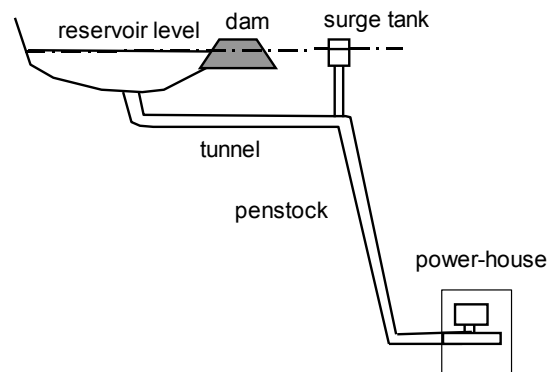


Figure 5 : a schematic turbine station

The reach.

The reach is the part of the river or of the channel separated by power plants and dams (part colored in

¹The head H is the difference of altitude between the level in the reservoir and at the turbine admission.

²A rough expression is $Pe = k \cdot H \cdot Qe$, where k is a factor that depends on the efficiency. A typical value is $k = 8 \text{ kJ/m}^4$

gray in Figure 6). The power plants can be located beside the dam, see the power plant 2 in the figure, or in a diversion like for the power plant 1. The reach can be several km long and its dynamics can be rather slow and delayed. Although the reach is not a reservoir it has a storage capacity named the retention volume that corresponds to the change of the water profile along the flow axis.

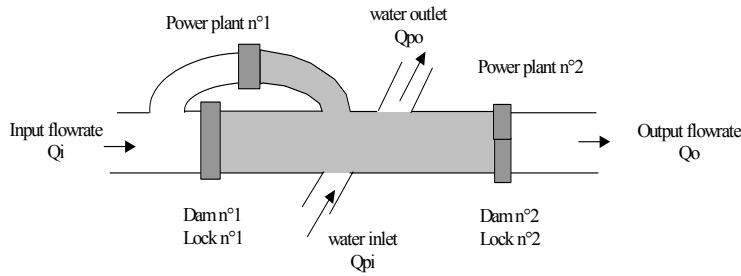


Figure 6 : schematic of a river reach

2.1.2 The Valley

An Hydro Power Valley is an interconnection of several reaches and storages. Every HPV is a special case because of the geography : the differences comes from the river and lake characteristics, the other water users that impose their constraints (irrigation, navigation, industrial and domestic use). However we can distinguish the cascade run-of-river plants (Rhine, Donau, Neckar, ..) and HPV that contains lake storage.

In Figure 7, we present a case study that will be used to evaluate a HD-MPC solution for HPV. The system consists of 5 storage lakes connected by a pump and turbines to a river. The river is composed of 3 reaches. The plant 3 has a reversible group that can either pump or turbine the water.

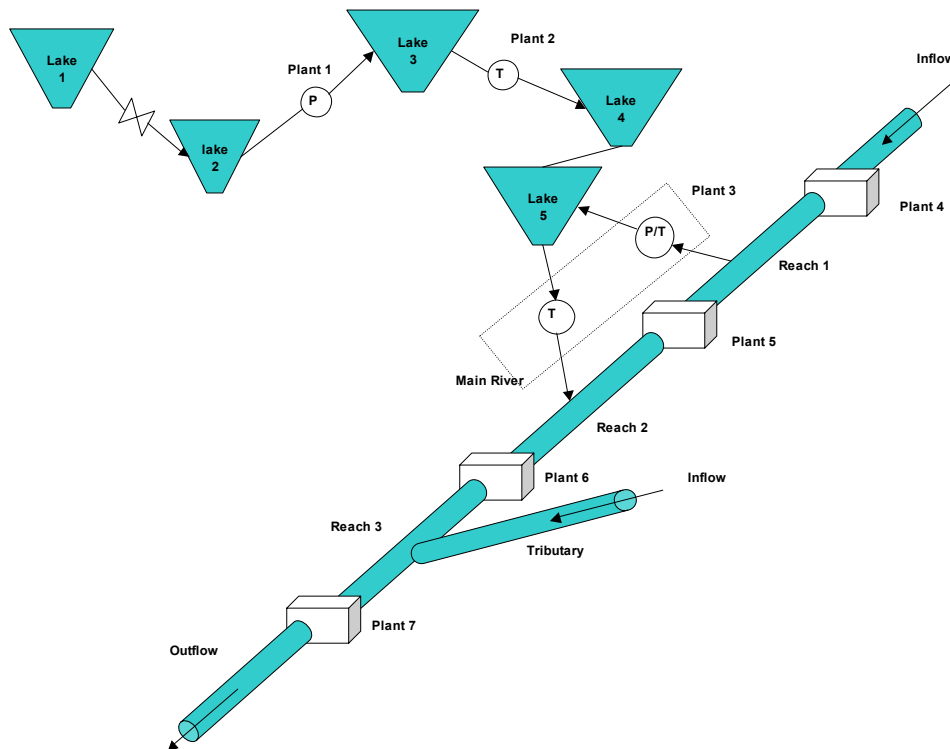


Figure 7 : Study Case for HPV HD-MPC application

Depending on the characteristics of the HPV we can define several modes :

Run-of-river :

This mode corresponds to the production without modulation. The main objective is to keep the levels

between given bounds. No constraints on the power are imposed. In general this lead to a quasi baseload power generation.

Power/Flow rate modulation :

This mode is available on HPV with storage capabilities and on big run-of-river HPV. The power plant can make power variations during a given period either by a modification of the flow rate or of the power set-point. This variation can be imposed remotely by the Transmission System Operator (TSO) or can be launched on a time basis. In a power valley some plants are dedicated to power modulation (plant 3 in Figure 7). The other plants downstream the river have to mitigate the effect of the power modulation. For instance the Plant 7 is designated as a demodulation station.

Power control mode :

The electricity can't be stored and every power plant of a sufficient size must participate to the frequency control. The frequency is indeed a variable that characterizes the Production/Demand equilibrium in a power system : when the production is too low the network frequency decreases, and whereas the production is too high, the frequency increases. UCTE imposes the participation to the frequency control with a power set-point P equals to the sum of 3 terms $P_o + k DF + N Pr$.

P_o is the daily program that can be rescheduled during the day every hours. The program P_o can't take into account all the perturbations of the power system between two rescheduling.

The primary frequency control $k DF$ is needed to stabilize the frequency. The participation coefficient k depends on the dynamical capabilities of the plant. An important point is indeed the dynamic of the response. If a plant declares a primary reserve RPM to the network operator 90 % of the reserve must be delivered in 30 seconds for a frequency step variation of 0.2 Hz and the reserve must be maintained during 15 min. The local primary frequency controls don't fulfill the frequency regulation at a 50 Hz set-point.

Figure 8 gives an example of power response that satisfies the criteria. The overshoot at the beginning is not necessary present.

The frequency regulation at 50 Hz is achieved by a centralized control that calculates a secondary signal N that lies between -1 and $+1$. On a ramp variation in 800s of the whole range, the power response must follow the set-point variation $N Pr$ with a maximum error of 15 %.

Figure 9 gives an example of power response that satisfies the criteria. Pr is defined as the secondary frequency response participation and is declared at the TSO.

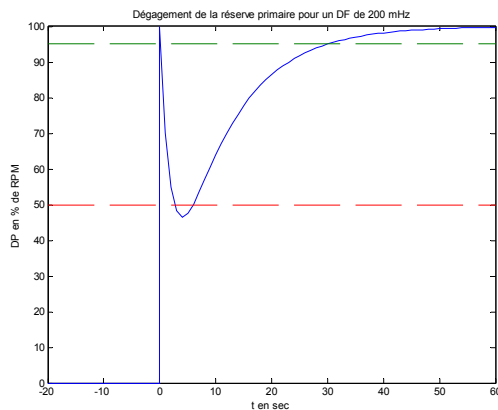


Figure 8 :Primary frequency response

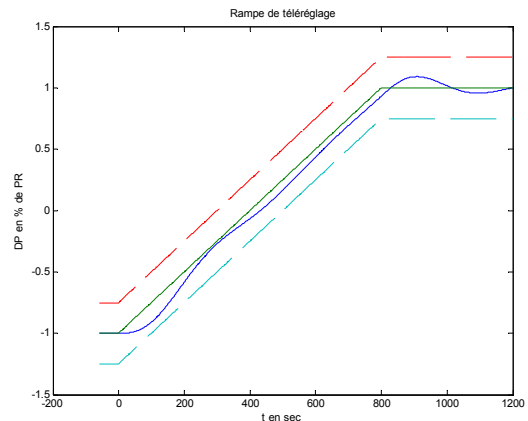


Figure 9 : Secondary frequency response

Flood management mode :

In case of flood, the HPV controls are quite different and some control loops are activated like the discharge valves of the dam. In that case, the goal of the HPV control is to mitigate the impact of power plants on flood. In other words, the power plants shouldn't make the things worse.

An HPV can be piloted following the different modes. For each mode the power production must at all time respect the constraints that mainly concerns the water levels at given points of the valley. These level set-points can stem from several other uses of the valley (navigation, tourism, irrigation, industry). Constraints on flow rate must also be met to ensure an easy navigation for the barges, a sufficient flow in the natural river for the flora and the fauna, and sufficient water for agriculture and industry. Other constraints are imposed by the machines (pumps and turbine). Those concern mainly the flow rate, the flow rate variation or the power and the power variations. From a practical point of view, the production program P_o required by the TSO is tested by the operator to check that all the constraints are satisfied. If not, a modified program is proposed to the TSO.

2.2 Control description

The control of the hydro power valley is done through a Hierarchical and Distributed control system that consists of 3 layers as defined in Figure 10.

- The rank 3 Control System is a SCADA located in the HPV control room. The operator is piloting the valley through its HMI that manages the alarms. Besides a communication with the plants of the valley, it is connected to the Grid operator (TSO). The operator has got off-line simulators to check that the program imposed by the TSO respects the constraints.
- The rank 2 consists of control system located in the powerhouse. These control systems manage the level and flow control, the secondary frequency control as well as secondary voltage control. In general a power plant is composed of several units that can be launched sequentially. The rank 2 control system defines the dispatch of each unit to optimize the efficiency of the total plant. The unit commitment takes into account the priorities of each unit, the startup time and efficiency of each unit, as well as the power and power rate limitations.
- The rank 1 control system is dedicated to one sole unit. It contains safety functions, and start-up and shutdown sequences. There is also rank 1 control system dedicated to each dam.
- The rank 0 consists of the local measurement and local control of one unit that are not implemented in the rank 1 PLC. Speed and voltage belong to the rank 0. As these loops are very fast, they are generally implemented in specific control system.

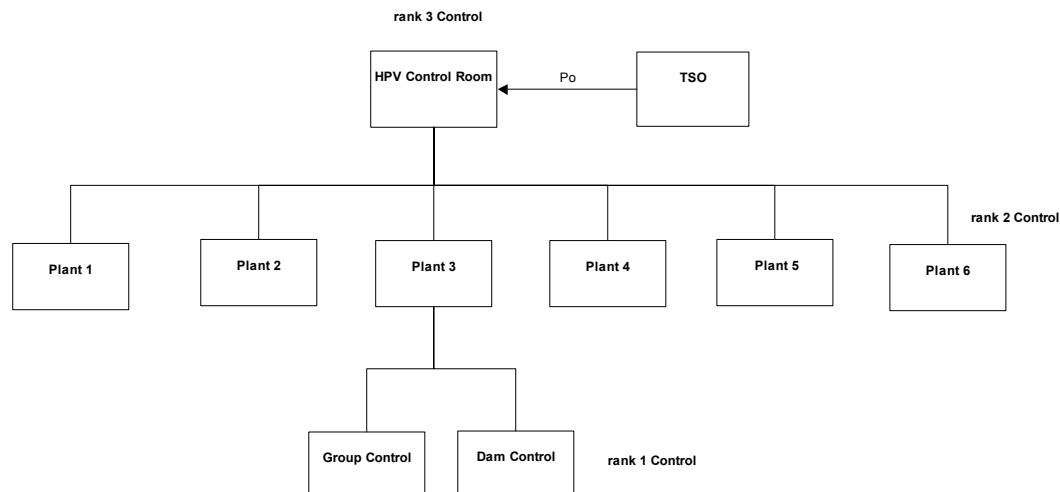


Figure 10 : hierarchical and Distributed control System

The Figure 11 presents the main control loops that apply on a power plant and that are implemented in the different ranks of the control architecture. This schema is general and has to be instantiated following the type of power plant : for lake, the level is not regulated, for run-of-river, the level or the power can be regulated.

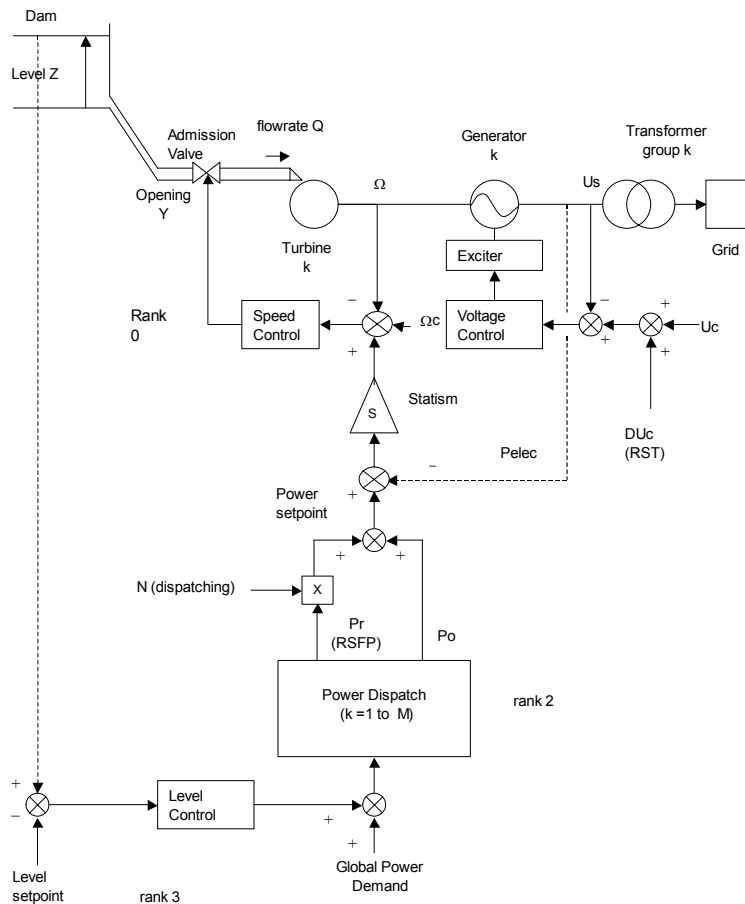


Figure 11 : power plant control .

The rank 3 imposes the power demand and the level set-point. In fact with only one control mean (opening of the turbine inlet) it is not possible to regulate both variables without steady state error.

In practice the power demand is therefore corrected by the output of the level controller. In fact the sole requirement for the level is to stay between limits. With classical control this requirement is achieved by either adding a dead-zone in the level controller or by computing a level set-point coherent with the power demand so that the power regulation is not biased with the level control.

Besides the power controller, the other elements that influence the level of the lakes and reaches are the perturbation inflows due to the streams, rains and thaws.

3 Literature Survey on modelling and control

In this section we list some papers on modeling and control of hydro system and of cascaded river power plants.

- [1] M.v Siebenthal, G.Glanzmann : Supervisory Water Level Control for Cascaded River Power Plants, Master Thesis, Automatic Control Laboratory, ETH Zurich, 2004
- [2] E.Gullhamn : Control of Water Content and Retention in Hydropower Plant Cascades, MS, KTH Stockholm , 2004
- [3] T. Rauschenbach: Eine allgemeingültige Methode zur Modelierung und Mehrikriteriellen optimalen Führung von Staustufen und Staustufenkaskaden, VDI Verlag 1999.
- [4] Fabienne Kein, Identification et commande robuste appliquées à la gestion automatisée des aménagements hydrauliques, Thèse Paris Orsay, 1992.
- [5] C. Setz, A. Heinrich, P. Rostalski, G. Papafotiou, M.Morari. Application of Model Predictive Control to a cascade of river power plants. Proceedings of the 17th World Congress. The International Federation of Automatic Control. Seoul, Korea, July 6-11, 2008.

- [4] D. Van Mien, F. Klein, Robust control for hydraulic power plants, 1st IEEE Conf. On Control Application, Dayton, 1992
- [6] D. Dumur, A. Libaux, P. Boucher, Robust RST control for «Basse-Isère» run-of-river cascaded hydro-electric plants, Conf. ECC 2001.
- [7] X. Litrico, D. George : Robust continuous-time and discrete-time flow control of a dam-river system. (I) Modelling, Applied Mathematical Modelling 23 (1999) 809-827
- [8] X. Litrico, D. George : Robust continuous-time and discrete-time flow control of a dam-river system. (II) Controller design, Applied Mathematical Modelling 23 (1999) 829-846

The document [4] is a thesis on robust control applied to the Rhine river. The models used for the control design are identified transfer functions obtained with simulated data given by detailed model implementing the Barré Saint-Venant equations. The polynomial LQG/LTR approach is tested on the level control. Centralized control obtained with multivariable state space LQG/LTR technique are done on a cascade of ten plants. The results obtained are compared with a decentralized control with serial and parallel anticipation and gives similar results.

Model Predictive Control of cascade plants are given in the report [1, 2]. The MPC is centralized and the estimator is decentralized. The model used for the control is a linear and discretized Saint-Venant equation. A model reduction has been used to diminish the model order. The solution has been compared with decentralized control consisting of PI controller with a feedforward term.

The document [3] presents a coordinator using Fuzzy control for a cascade river plants. The solution is tested in simulation with a simulink model that implement a discretized version of Saint Venant equation.

The articles [7,8] propose a 2 dimensional transfer function obtained by applying a Laplace transform on a linearized Saint Venant equations. This transfer function can be used for robust frequential method (PID, H_∞). We haven't found any use of these models for MPC control.

4 HD-MPC Control Specification for HPV

In this section the general specifications for the HPV study case shown in Figure 7 are presented. After a description of the process under consideration, the objective functions and the constraints are given. Validation tests in simulation to assess the robustness of the HD-MPC in real conditions are proposed at the end of the section.

4.1 Process description

The global system shown in Figure 12 is divided in 9 subsystems (P1 to P9) that are interconnected together and correspond to the following definitions :

- P1 : Lake1 + Valve 1.
- P2 : Lake 2 + Plant 1.
- P3 : Lake 3 + Plant 2.
- P4 : Lake 4. As P4 has no control input it will be probably necessary to merge it with another subsystem (P5 for instance)
- P5 : Lake 5 + Plant 3. This system has 2 control inputs corresponding to the pump and turbine. The control input that corresponds to the pump-turbine plant (u_{57} in Figure 12) can take positive and negative values.
- P6 : Plant 4.
- P7 : Plant 5 + Reach 1.
- P8 : Plant 6 + Reach 2.
- P9 : Plant 7 + Reach 3.

The connection between the subsystems is given in Figure 12. In this figure as in the whole document, the following notation is adopted :

For each subsystem i noted P_i :

- L_i is the level;
- P_{ei} is the algebraic electrical power (negative if the plant consumes, positive if it produces);

- d_i is the water inflow perturbation;
- u_i is the control input that's the discharge reference;
- q_{ij} is the algebraic flow rate from subsystem P_i to subsystem P_j

The further assumptions are made :

- each power plant will be seen as a unit. We suppose that the dispatch between the different turbines is imposed and has not to be optimized by the HD-MPC algorithm.
- At the rank 2 and rank 3 levels, the fast dynamics of the turbine can be neglected and the control input u_i be considered equal to the discharge flowrate q_{oi} .

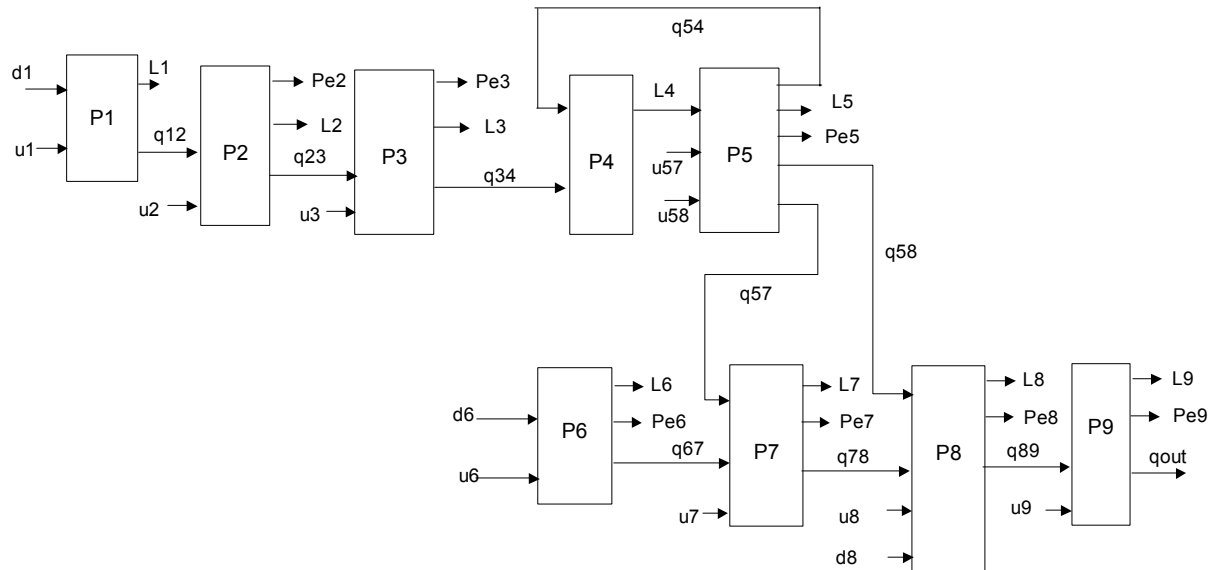


Figure 12 : HPV Functional Diagram

4.2 Objective functions

The global objective for the valley depends on the type of operating mode and on the context of the optimization.

- Basically, for run-of-river mode the objective function is to maximize the global power production while respecting the constraints.
- For power control mode or power/flow modulation the objective is on the contrary to minimize the power regulation error deviation by respecting the constraints.

The objective function depends also on the context of the optimization.

- Each day before 4 p.m a program is actually proposed for the next day. This program is sent to the power fleet manager that integrates it in his global power optimization. During this preparation phase the objective of the HPV can be the maximization of the energy produced.
- However in the realization phase, the objective is different, the load is imposed by the grid manager and the goal of the control is to follow as much as possible the program while respecting the constraints even if it can produce more.

The following objective functions will be envisaged for the HPV to cover the preparation and realization phase.

Objective 1 : Maximization of the produced energy.

The power produced can be weighted by a electricity cost $ce(t)$. The criterion J_1 is additive and can be decomposed on the M power plants of the HPV as it will be indicated in the next section. H is the optimization horizon that is in general a day. The optimization will find the control sequence u that

maximizes $J1(u, x_0)$ with x_0 being the initial condition.

$$J1(u, x_0) = \int_{t=0}^H ce(t).Pe(t)dt = \int_{t=0}^H \sum_{i=1}^M ce(t).Pei(t)dt = \sum_{i=1}^M \int_{t=0}^H ce(t).Pei(t)dt \quad (1)$$

Objective 2 : Minimization of the power regulation error.

In this regulation problem the goal is to follow the demand $Pd(t)$ as close as possible, that is to minimize $J2$. The norm used is generally the 2-norm. The horizon is in general shorter than the one used for the day optimization but must be sufficiently long to take all the delayed effects into account. The objective function $J2$ is unfortunately not additive and can not be dispatched over the M power plants.

$$J2(u, x_0) = \int_{t=0}^H \|Pe(t) - Pd(t)\| dt = \int_{t=0}^H \|Pd(t) - \sum_{i=1}^M Pei(t)\| dt \quad (2)$$

Objective 3 : Quadratic regulation problem.

We suppose that the trajectory for the power and level have been defined by an off-line optimization. The program following can then be defined by $J3$ which is a LQ like problem. In that case the function is additive and a decomposition over the M subsystem is possible.

$$J3(u, x_0) = \int_{t=0}^H \sum_{i=1}^M Qi \|Li(t) - Lref(t)\|^2 + Ri \|ui(t)\|^2 dt \quad (3)$$

The problem $J1$ to $J3$ are formulated in continuous time. Analog discrete-time objective function can be defined with a summation instead of an integral operator.

4.3 Constraints functions.

The control that minimizes the objective functions must respect several operating constraints. These constraints can be classified in global and local constraints. Local constraints are equalities or inequalities that applied on variables of only one subsystem. Global constraints are equalities or inequalities that applied on variables belonging to several subsystems. We will give continuous expressions for the constraints. Discrete version will be derived if needed.

4.3.1 Local constraints

The local inequalities constraints .

The constraints given by equation 4 are upper and lower limits for the lake and reach levels. These constraints will be more stringent if the storage capacity is less.

The constraints equations 5 and 6 are limitations for the discharge flow rates. These limitations can come from the turbine itself or from constraints on the flow variations downstream the power plants.

In the optimization we can plane the authorized interval to keep a maneuverability margin. Playing with the limits can be a simple method to obtain a nominal solution that works also for off-design cases.

$$\underline{Li} \leq Li \leq \bar{Li} \quad (4)$$

$$\underline{ui} \leq ui \leq \bar{ui} \quad (5)$$

$$\underline{vi} \leq dui/dt \leq \bar{vi} \quad (6)$$

The local equality constraints .

The local equality constraints correspond to the process equation 7 that can be more or less detailed (we suppose that these equation can be formulated as a DAE). For flow in penstocks, where the dynamics are fast, algebraic equations can be sufficient. On the other hand, for reaches and lakes, dynamic models are needed to reproduce the storage and the delay effects. Depending on the desired precision and on the domain of validity, these models correspond to ordinary differential equations (linear or not) or to partial differential equations (linear or not). The form of the equations kept for the simulation and the optimization will be defined in a next report. For storage lake a supplementary constraints concerning the quantity of water that can be used during the period are imposed. It can be a constraint on the integral of the discharge or a terminal constraints on the level (see equation 8).

$$f(x_i, dx_i/dt, u_i) = 0 \quad (7)$$

$$\int_0^H u_i(t) dt = V_i \text{ or } L_i(H) = L_{final} \quad (8)$$

4.3.2 global constraints.

The main global constraints are the mass conservation between the subsystems.

It corresponds to equation 9 where u_i is the discharge of the subsystem i and q_{ij} is the flow to the subsystem j . In the considered HPV these equations can be easily obtained from the network description.

$$u_i = q_{ij} \quad (9)$$

Remarks :

- In the local problem resulting from the decomposition, the subsystem j will have $1+n_j$ inputs where n_j is the number of flow rates coming from the other subsystems. For the study case the local inputs are shown in Figure 12.
- For the subsystem 5, Figure 12 gives 3 output flow rates to subsystem 4,7,8. Only the 2 last can be controlled by the discharges u_{57} et u_{58} . The flow rate q_{54} depends only on the gravity by equation 9. The partition is perhaps not the best. The global constraint equation (9) is indeed non differentiable and the subsystem P4 in that case has no control input. A merged subsystem P45 can be considered to remedy these problems. With this new partition the equation (9) will then be a local equation, which may facilitate the convergence of the coordination algorithm. Going further, we could ask ourselves if we can't simply aggregate the two lakes in a big reservoir with a storage volume that is the sum of them. In fact, the flowrate q_{57} is limited by the duct capacity and has to be modeled in order to correctly take its induced limitation on the discharge q_{58} and q_{57} .

$$q_{54} = \text{sign}(L_4 - L_5) D \sqrt{g |L_4 - L_5|} \quad (10)$$

4.4 Robustness issues

The objective functions given in §4.2 consider deterministic expressions for a nominal case.

In the real world, the perturbations can't be foreseen exactly and the model used will only be an approximation of the reality. The HD-MPC must be valid in spite of uncertainty in the perturbation and the design model.

4.4.1 Perturbation uncertainties

The HPV is a system submitted to perturbations that are not exactly known. The water inflows for the river can't be exactly predicted. In case of ancillary services, the power demand can be modified by the primary and secondary frequency response that is not known in advance.

MPC and receding horizon is normally able to deal with unknown perturbations. But we can wonder whether a better solution can be found by integrating random perturbations in the optimization scheme. At the present time, a clear vision of what should be done is not available and just search directions are given below. To deal with the random perturbations, we could take the expectation of the objective function over the different possible realizations. This problem can be very hard to solve in the general case. A simpler approach could be to optimize the objective function J for several scenarios s , where p_s is the probability of the scenario s .

$$J(u, x_0) = \sum_{s \in \Sigma} p_s J_s \quad (11)$$

The objective function (11) is additive and a hierarchical optimization can be envisaged with local optimizations that minimize J_s for a given scenario s and a coordinator that imposes constraints on the obtained solutions, for instance, the equality of the first control inputs $u_{s_i}(1) = u_{s_j}(1), \forall (s_i, s_j) \in \Sigma$

The choice of the objective function to adopt for the stochastic problem is still open and other solutions can be proposed like for instance the worst-case optimization that corresponds to the min-max problem

$$\min_{u \in U} \max_{s \in \Sigma} J_s.$$

4.4.2 Model uncertainties

Robustness requirements can be included in the objective functions. As for the perturbations multi model optimization can be considered (equation 11) and a coordination level that imposes a coherence of the solutions.

The neglected dynamics are another source of uncertainty. In the optimization the dynamics of the turbines will be neglected and the discharges will be considered equal to the control inputs. The electric power also depends on the discharges with a static relation. Frequency weighted criterion (for J_3 for instance) can be considered to improve robustness of the control in presence of un-modeled high frequency dynamics.

As for perturbations we can also trust the intrinsic robustness of the MPC scheme stemming from the receding horizon scheme. For parametric and non-parametric uncertainties, a simple solution is therefore to consider the optimization for the nominal case and to check afterwards the robustness of the MPC solution for the off-design cases.

4.5 Test Scenarios

Because of the uncertainties, the HPV HD-MPC algorithm has to be tested in simulation before a field experiment. The advantage to work with a model is that we can envisage exhaustive testing to assess the performance and the robustness of the solution.

The model used for the simulation may be finer than the model used for the control. The parameter of the control model will in that case be identified with data obtained with the non linear model. The former is an approximation of the latter so a bias will exist after identification. Non linear simulation in nominal conditions will show the effect of the model approximation on the results.

4.5.1 Nominal tests

The solution will be tested in design condition :

- inflow , power demand perfectly known;
- control and simulation model identical;

4.5.2 Robustness tests

A next step will be to see the impact of the model uncertainty on the solution:

- inflow, power demand perfectly known;
- control model and simulation model different (use of non linear Saint-Venant Equations). Different non linear models can be obtained by modifying the friction coefficients or the geometry (width, bottom, slope) of the reaches;
- model perturbations will be added for the discharge to represent flowrate measurement error, and dynamics of the turbine.

4.5.3 Off-design tests

The last validation tests will consider off-design condition :

- The power demand will be modified by an additive term that corresponds to the frequency response (this is an off-design condition if we apply a deterministic HD-MPC); Results can be compared with stochastic optimization results if available.
- Inflow uncertainties will be added; Results can be compared with stochastic optimization results if available.
- Communication faults between the local control and the coordination will be tested. In practice these events will probably be taken into account by the interlock implemented in the DCS. However, it could be interesting to see how a distributed control continues to work with a partial coordination.

5 Hierarchical Decomposition of control

5.1 HD-MPC architecture

In this section we propose several hierarchical and distributed schemes for the HPV valley. These are only suggestions that will not be necessarily implemented in the project.

A possible HPV HD-MPC architecture is given in Figure 13. It consists of 2 layers the "HPV Optimization" and the local controllers R_i layer. The local controllers R_i may be MPC or the existing PID controllers. These controllers are located in the power house in order to keep the control operational in case of communication loss between the HPV control room and the power house. The upper layer called the "HPV optimization" module will be located in the HPV control room.

The processing at this level will depend on the method used to decompose the global optimization problem.

Depending on the considered problem we can use different kinds of decomposition. Price, Quantity and Prediction decomposition correspond to a spatial partition, whereas cascade decomposition refers to temporal decomposition.

Price decomposition: The "HPV optimization" sends a price vector (associated to the global constraints) to the plants, and each of these subsystems minimizes a cost function that depends on those prices.

Quantity decomposition: The "HPV optimization" manages the constraints and sends the production set-points to the subsystems. Each subsystem minimizes its cost function and sends a price to the coordination. This approach is dual of the price one.

Prediction decomposition: Each subsystem deals with part of the coupling constraints and sends to the other ones a price associated with these constraints, and every subsystem takes into account the prices associated with the constraints that it does not treat.

Cascade decomposition: Two loop levels can be considered, a fast loop that regulates the variables around the set-points and manages the physical constraints, and an optimization loop that computes the set-points.

Here are some of the references on decomposition methods.

- [1] G. Cohen. Optimization with an auxiliary constraint and decomposition. SIAM Journal on control and optimization, 28(1):137-157, January 1990.
- [2] A. Rantzer. Dynamic Dual decomposition for Distributed Control. To appear at American Control Conference.2009.
- [3] M. Jamshidi. Large-Scale Systems: Modeling and Control. Elsevier Science Ltd. 1983.

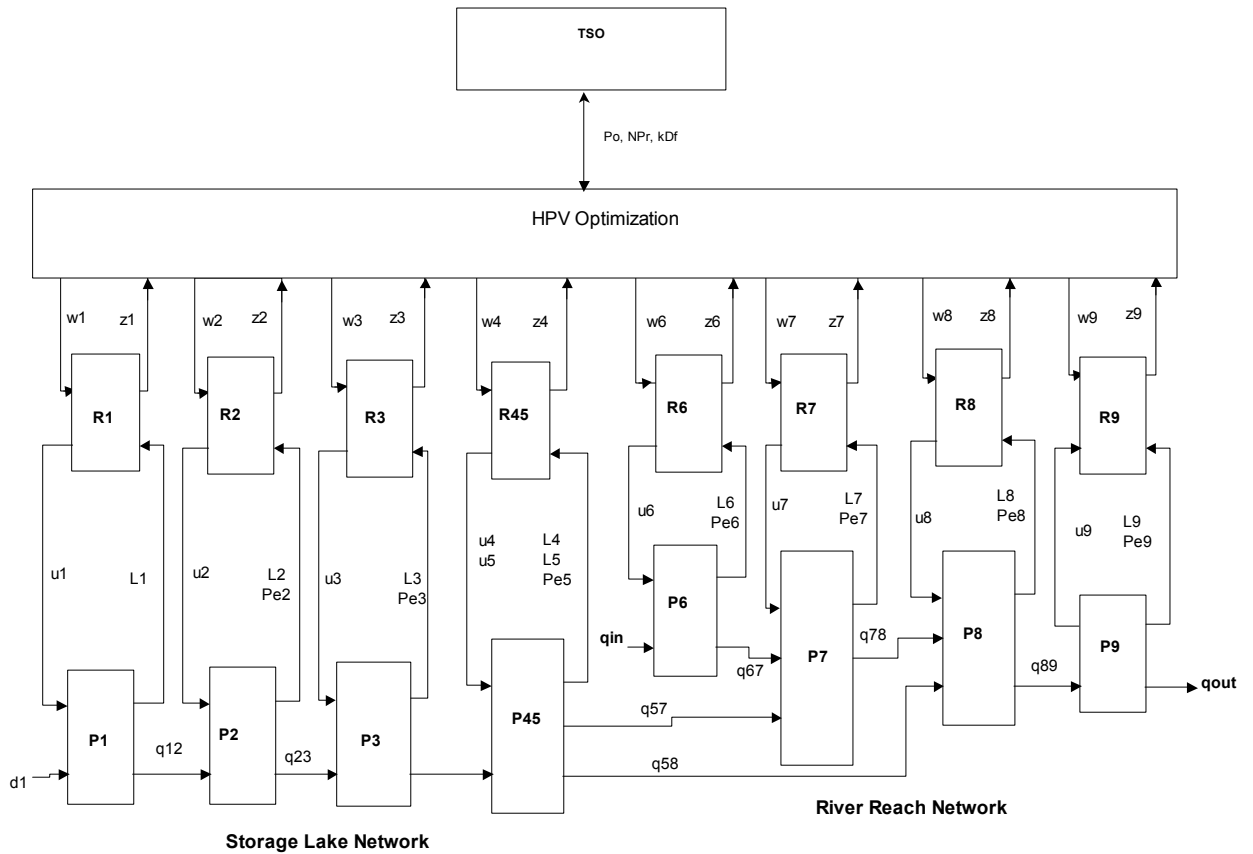


Figure 13 : HPV HD-MPC architecture

Remarks :

- In the Figure 13 the local controller R_i exchanges information with the upper-level (w_i, z_i). The nature of these signal depends on the type of coordination that will be chosen according to its performance in term of convergence. Theoretical developments are therefore needed to clarify the conditions for the convergence of distributed optimization in a MPC context. These conditions will probably imply restrictions on the possible models and objective functions. But even in a favorable case, the convergence may be rather slow and a key point will probably be the speed of convergence of the distributed optimization. For cascaded MPC scheme the conditions for a good convergence of the two loops will probably also imply restrictions on the type of model and objective function to use.
- In practice all the information needed for optimization should be available in the HPV control room and thus all the optimizations can be done remotely. The only requirement is that the “HPV optimization” will effectively send signals to the local controllers and that these local controllers must properly work even in case of loss of communication. After a remote optimization, level and power set-points can be computed and sent to local controller instead of the coordination signals (w, z). Remote optimization present the advantage to save a lot of data transfer and to improve the robustness in case of communication loss.

5.2 Definitions & assumptions

5.2.1 Primal and dual problem

We assume that the global control problem is formulated by the constrained optimization problem :

$$u^* = \arg \min_{u, q} J(q, u) \tag{12}$$

$$g(q, u) = 0 \tag{13}$$

q are the input flow-rates for each subsystem and u the discharge control as indicated in Figure 12. We consider only the global constraints like equation 9.

Remark : The local constraints won't be taken into account for simplicity and because the main point of interest in this section is the coordination mechanism. These local constraints would correspond to process equation and domain constraints and could be embedded after discretization in equation as :

$$g_i(q_{ji}, u_i) = 0$$

The primal problem (Equation 12-13) can be associated to the dual one by applying lagrangian relaxation, that is the maximization of the dual function $\Psi(\lambda)$)

$$\max_{\lambda_j} \Psi(\lambda) \quad (14)$$

$$\Psi(\lambda) = \min_{u, q} \{ J(q, u) + \lambda g(q, u) \}, \text{ where } \lambda \text{ and } g(q, u) \text{ are column and line vector.} \quad (15)$$

5.2.2 Additivity

We suppose that the objective and constraint functions are additive :

$$J(q, u) = \sum_{i=1}^M J_i(q_{ji}, u_i) \quad (16)$$

$$g(q, u) = \sum_{i=1}^M g_i(q_{ij}, u_i) \quad (17)$$

where M is the number of subsystems.

Remarks

- The additive decomposition assumption is not verified for all the objective functions listed in §4.2.
- Let 2 subsystems 1 and 2 connected serially. The discharge of subsystem 1 is the input flow rate for the subsystem 2. This connection corresponds to the following equality constraint and decomposition.

$$g(q, u) = u_1 - q_{12} = 0$$

$$g_1(q_{21}, u_1) = u_1$$

$$g_2(q_{12}, u_2) = -q_{12}$$

5.3 Price decomposition

The price decomposition aims to solve the dual problem (14-17) rewritten in (18).

$$J(q^*, u^*) = \max_{\lambda} [\min_{u_i, q} \sum_{i=1}^M J_i(q_{ji}, u_i) + \lambda g_i(q_{ij}, u_i)] \quad (18)$$

where q is a vector which contains q_{ij} and q_{ji} .

For a price λ , each local controller solves the local problem.

$$[u_i^*, q^*] = \arg \min_{u_i, q} J_i(q_{ji}, u_i) + \lambda g_i(q_{ij}, u_i)$$

The local problem is in fact a constrained problem itself, but local constraints are ignored for simplicity.

With the new solution for the local problem, the price is adapted with the following equation :

$$\lambda^* = \lambda + \varepsilon \sum_{i=1}^M g_i(q_{ij}^*, u_i^*) \quad (19)$$

The optimal value u_i and λ are not obtained in a first attempt. So u_i and q_{ij} have to be recalculated with the new λ and so on until the new value of λ is equal to the previous one. In that case the equality constraint is satisfied. The solution is admissible (respects the constraints) only at the convergence.

5.4 Quantity decomposition

For a quantity decomposition, the term of the constraint equations (equation 17) are considered as quantities θ_i . The upper level imposes values for the quantities such that the constraint equality is fulfilled.

$$g(q,u) = \sum_{i=1}^M g_i(q_{ij},u_i) = \sum_{i=1}^M \theta_i = 0 \quad (20)$$

For given quantities, the local constrained problems are solved:

$$\min_{u_i, q_{ji}} \sum_{i=1}^M J_i(q_{ji}, u_i) \quad (21)$$

$$g_i(q_{ij}, u_i) = \theta_i \quad (22)$$

The local problems can be solved by Lagrangian relaxation with the maximization of the dual function (23).

$$[u_i^*, q^*, \lambda_i^*] = \arg \max_{\lambda_i} \min_{u_i, q} \{ J_i(q_{ji}, u_i) + \lambda_i (g_i(q_{ij}, u_i) - \theta_i) \} \quad (23)$$

The coordinator is updating the quantity with the price given by the local optimization

$$\theta_i^* = \theta_i + \varepsilon \left(\lambda_i - \frac{1}{M} \sum_{j=1}^M \lambda_j^* \right) \quad (24)$$

Again as for price decomposition the update does not give the optimal solution in a first iteration. And new quantities have to be imposed to the local problem until the convergence is reached.

If the local optimizations succeed, then the solutions meet the constraints even if the optimum is not reached. This could be a advantage over price decomposition where the constraints are satisfied at optimum. But, The local optimization is a constrained and more complicate problem without solution in some cases (the plant is not able to give the quantity by respecting its local constraints).

5.5 Prediction decomposition

Unlike price and quantity decomposition, Interaction Prediction Principle is a method that was originally developed for control purpose. Each subsystem is responsible of a subset of the constraints This subset can be for instance the constraints equations with its neighbors given by the equation 9 restricted to variables of the subsystem i (u_i, q_{ji}, q_{ij}). We keep the notation g_i for the constraints functions but it represents in fact a restriction on the equations kept for the subsystem i .

We will note the constraints equations $g_i(u_i, q_{ij})$, that are taken into account by subsystem i .

We define also μ_j is the price the other subsystems are willing to pay to the system i to fulfill their own constraints. The contributions of system i to the constraints of the subsystem j will be noted $g_j(u_i, q_{ij})$.

We suppose that we know the interaction variables q_{ij} and the price μ_j given by the subsystem j to the subsystem i . Then, for subsystem i the local problem corresponds to the equations (25) and (26), where a term is added to the objective function to take into account the effect of the local control on the constraints for the j th subsystem. This problem is equivalent under certain conditions to be defined to the lagrangian augmented problem (27).

$$\min_{u_i} \{ J_i(q_{ji}, u_i) + \sum_{j \neq i} \mu_j g_j(u_i, q_{ij}) \} \quad (25)$$

$$g_i(u_i, q_{ji}) = 0 \quad (26)$$

$$[u_i^*, q_{ji}^*, \mu_i^*] = \arg \max_{\mu_i} \min_{u_i, q_{ji}} \{ \{ J_i(q_{ji}, u_i) + \sum_{j \neq i} \mu_j g_j(u_i, q_{ij}) \} + \mu_i g_i(u_i, q_{ji}) \} \quad (27)$$

The solution to problem (27) gives updates for the prices μ_i and the interaction variables q_{ji} that are sent to the subsystem j . In parallel, the j th optimization will compute the new price μ_j and interaction q_{ji} to be sent to the i th subsystem. At convergence the prices must be equal. With this coordination, the

exchanges are done on a one-on-one basis. No decision are taken outside the local optimization. The upper level described in Figure 13 is just a communication level.

5.6 Temporal or Cascade decomposition

Some of the objective functions presented in the §4.2 are not additive and can't be decomposed over the M subsystem. In that case, a cascade decomposition can be implemented.

The "HPV optimization" module solves a centralized problem with a simplified model that considers only the slow dynamics. This optimization computes the power and level setpoints for each subsystems.

The lower MPC controller or PID applies these set-point on the real system that will respond differently.

The receding horizon technique mitigates the drift due to the model approximation.

To apply this decomposition a method is needed to split the dynamics of the system in fast and slow dynamics. Singular perturbations reduction technique may be a solution. Identification is also a way to derive reduced order model. As the system is non linear, non linear low order model are desired as state affine model for instance.

6 Conclusion

The report present the HPV study case that will be considered in the HD-MPC project. The valley consist of a network of lakes connected to a river composed of several reaches.

The objective is to optimize the power production of the valley while respecting all the operating constraints imposed for safety or environmental reason.

The report gives the control requirement objective functions, constraints and robustness.

Several possible objective functions are proposed. Some of them can be decomposed over the subsystems, other can't. The constraints are classified in local and global constraints. Only the latter have to be taken into account in the coordinator. Robustness is very important feature, because the models are only approximation and because of the unknown perturbations.

A architecture is proposed with several coordination approach. Spatial decomposition can be applied with additive problem. The objective function and the constraints function can be decomposed in a sum of term corresponding to each subsystems. Three main decomposition have been formulated for the HPV problem (price decomposition, quantity decomposition, and the so-called Interaction Prediction Principle). For temporal decomposition, the idea is to separate the model in fast and slow parts and optimize the two parts in a cascade manner. The slow part gives reference to the fast part.

The next step in the project is to develop model for control and for validation. Algorithm will be developed and tested afterwards. Several test scenarios in nominal and off-design case have been proposed to assess the performance and the robustness in a real case. A key feature will be the speed of convergence of the algorithm and the guaranty for the control to perform correctly even if the convergence is not reached or in case of communication loss.