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Table of Content

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Executive Summary

The purpose of this report is to offer a survey of the most usual approaches optimisation methods for MPC of uncertain large-scale systems. A deep analysis of existing methods has been done, reviewing the main approaches: worst-case scenario, stochastic approach, randomized algorithms and measurement-based optimization. The conclusion is that it is a complex problem that has to deal with several aspects such as uncertainties description, constraint satisfaction, feasibility, stability, computation complexity, etc. All the presented methods show advantages and drawbacks; therefore new contributions to the problem can be expected in the project.

1 Introduction

The presence of uncertainty in the system description has been always recognized as a critical issue in control theory and applications. In practical situations, with uncertainty coming from model mismatch and process disturbances, it is not enough to determine numerically an optimal solution on the basis of a nominal model and apply it to the process to implement an optimization-based control strategy, such as Model Predictive Control. In contrast to such a nominal optimization approach, uncertainty has to be considered explicitly.

Since the early 1980s, several approaches based on a direct characterization of the uncertainty into the plant have been proposed. The design objective hence becomes the computation of a controller that is guaranteed to perform satisfactorily against all possible uncertainty realizations, thus leading to a worst-case (or robust) solution. The purpose of this report is to offer a survey of the most usual approaches optimisation methods for MPC of uncertain large-scale systems. Standard MPC algorithms, however, do not take directly into account model uncertainties and disturbances. Although the feedback mechanism itself is able to partially compensate for them, robust control designs that cope with uncertainties in an explicit way are of interest in modern MPC theory. In this report, the implementation and architecture details will not be discussed deeply, as this is the scope of other reports.

In the absence of measurements, a solution is sought that can take the uncertainty into account explicitly. The uncertainty is dealt with by considering several possible values for the uncertain parameters. The optimization is performed either by considering the ‘worst-case scenario’, where the optimal solution sought has to minimize the objective for the most critical set of parameter combinations, or in an ‘expected sense’, where the probabilistic constraints are satisfied on average.

However, this typically requires solving a much more complex optimization problem that includes many more differential and inequality constraints than without uncertainty [18, 27]. The resulting solution is conservative for the worst-case scenario or does not guarantee to satisfy the constraints in the probabilistic setting, but it corresponds to the best strategy available in the absence of measurements.

The following sections describe some of the methods that are used to solve the optimisation problem in control systems.

2 Worst-case scenario

Many robust MPC schemes are based on the min-max strategy originally proposed in [54], where the performance index due to the worst possible disturbance realization is minimized. In the context of robust model predictive control (MPC), the problem was tackled by Campo and Morari [12]. In general, solving a min–max problem subject to constraints and bounded additive disturbances is computationally too demanding for practical implementation. Some approaches to overcome this complexity can be found in literature. Lee and Kouvaritakis proposed a linear programming approach in [28]. In [30], the worst case value of the objective function is bounded by means of a Linear Matrix Inequality (LMI). Langson et al. [27] presented a feedback model predictive control that maintains the trajectories in a tube. Min–max MPC can also be addressed by the use of multiparametric programming [9], [24], [32], [33]; however, as it is well known, these results are in general hard to apply to large scale systems. Early works deal with open-loop predictions and optimize a single sequence of control inputs for the worst possible trajectory of the uncertain variables. Further results address the feedback min–max problem, where the optimization is done over a sequence of control laws in order to take into account that more information about uncertain variables will be available in the future through feedback.

However, despite the amount of work developed in this field, few applications can be found in the literature, and moreover, they usually deal with small scale fast systems such as in [34]. One of the main reasons is that the computational complexity of this class of optimization problems grows exponentially with the prediction horizon and the size of the process. This problem is even more evident in the so-called feedback min-max control schemes.

3 Stochastic Approach

Moreover, in general, it is common feeling that the control laws are too conservative. Stochastic MPC takes a different route to solve MPC problems under uncertainty. By modelling the uncertainty as an stochastic variable, the expected value of the cost function is minimized. As in the min-max case, feedback predictions are taken into account (see [43]). The stochastic view of the disturbance in MPC could be traced back to Clarke's Generalized Predictive Control [15]. Like in many approaches that follow the same line of thinking, the results are valid only in the unconstrained case. Early works in SMPC deal with input constraints for different classes of models, see e.g. [48], [3]. However, state constraints are not tackled, and efficient algorithms for evaluating the control law are not provided.

More recently, robust MPC schemes that can be solved by Stochastic Programming (SP) techniques have been proposed [18, 35]. Stochastic Programming is a special class of mathematical programming that involves optimization under uncertainty (see [7], [25], [39]). Nowadays SP is becoming a mature theory that is successfully applied in several other application domains (see the survey [40]). For other contributions in control theory of SP techniques the reader is referred to [4], [25], [18]. From the computational viewpoint specific efficient algorithms for stochastic LP (Linear Programming) and QP (Quadratic Programming) are available in the literature (see for example [7], [44], [14], [38]) and commercial solutions to SP were announced recently [16].

There are several applications of SP to large scale problems, in particular to strategic level decision-making problems such as planning for investments or scheduling [13], [17], [21], [41], [5]. In this class of problems the uncertainties associated with long-range forecasts make these decision problems very complex. Stochastic programming is one of the most powerful analytical tools to support decision-making under uncertainty. However, there are still few results in the control community, in particular, because of the difficulties in proving any kind of guaranteed closed-loop properties such as stability or constraint satisfaction. The fact is that the development of stochastic controllers with guaranteed properties is still an open field.

4 Randomized algorithms

An alternative paradigm is to assume that the plant uncertainty is probabilistically described so that a randomized algorithm may be used to obtain, normally in polynomial time, a solution that satisfies some given properties [47], [49].

Uncertainty randomization is now widely accepted as an effective tool in dealing with control problems which are computationally difficult, see e.g. [47]. In particular, regarding synthesis of a controller to achieve a given performance, two complementary approaches, sequential and non-sequential, have been proposed in recent years.

For sequential methods, the resulting iterative algorithms are based on stochastic gradient [10], [19], [20], [37], ellipsoid iterations [23], [36] or analytic centre cutting plane methods [11], see also [2] for other classes of sequential algorithms. Convergence properties in finite-time are in fact one of the focal points of these papers. Various control problems have been solved using these sequential randomized algorithms, including robust LQ (Linear Quadratic) regulators, switched systems, and uncertain Linear Matrix Inequalities. Sequential methods are mostly used for convex problems; they are very useful because, at each iteration, the computational time is usually affordable. However, the number of iterations may be very large and depends on a stopping rule.

A classical approach for non-sequential methods is based upon statistical learning theory, see [50], [46] and [31] for further details. In particular, the use of this theory for feedback design of uncertain systems has been initiated in [51]; subsequent work along this direction include [52], [53]. However, the sample size bounds derived in these papers, which guarantee that the obtained solution meets a given probabilistic specification, may be too conservative for being practically useful in a system and control context if the available computational resources are limited. The advantage of these methods is that the problem under attention may be non-convex and can be solved in one-shot without any stopping rule.

5 Measurement-based optimization

Process measurements can be used to adapt the input trajectories in order to guarantee optimality despite the presence of uncertainty. This is typically done by on-line re-optimization of the dynamic optimization problem [1]. At each sampling time, the current states, which serve as initial conditions for the next optimization, are updated on the basis of process measurements. If necessary, the model parameters might also be updated using measurements. Furthermore, since all required process variables are seldom accessible through measurements, suitable estimation techniques are also necessary [29, 26]. Note that the various state and parameter updates and the re-optimization represent computationally involved tasks to be performed on-line. The solution of such problems is still a major challenge although significant progress has been made in recent years [6]. This approach is mainly used for dynamic plant optimization, typically in batch processes.

A conceptually different way of using measurements for on-line process optimization has been proposed recently by Srinivasan et al. [45]. In this work, a decentralized control scheme is proposed to track the Necessary Conditions of Optimality (NCO). Therefore, this approach is referred to as NCO tracking. It uses the concept of a solution model, which is a model that relates the various elements of the manipulated inputs to the different parts of the NCO. It is typically obtained by dissecting the input profiles obtained off-line using numerical optimization of a nominal process model and interpreting them by visual inspection. This approach requires a highly accurate numerical optimization procedure that can clearly reveal the different arcs in the optimal solution, and experience with and physical insight into the process. Recent work in numerical optimization of dynamic systems has shown that it is not only possible to compute highly accurate optimal solutions but also to determine their structural properties including the number, type and sequence of arcs as well as the determination of active path and terminal constraints [42]. A recent work [22] presents a systematic and automated approach to generate a solution model based on recent results in numerical optimization of dynamic systems. This concept provides the first step toward an entirely automated procedure for constrained dynamic optimization of uncertain large-scale processes.

6 Conclusions

The optimisation of uncertain large-scales systems has been a field of interest in the last twenty years and it there are still many open issues. It is a complex problem that has to deal with several aspects such as uncertainties description, constraint satisfaction, feasibility, stability, computation complexity, etc. There are several approaches to tackle the problem, some of which have been reviewed here. All the presented methods show advantages and drawbacks; therefore new contributions to the problem can be expected in the project.

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