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Author(s):	José García, Felipe Valencia, and Jairo Espinosa

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Project co-ordinator

Name: Bart De Schutter
Address: Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 Delft, The Netherlands
Phone Number: +31-15-2785113
Fax Number: +31-15-2786679
E-mail: b.deschutter@dcsc.tudelft.nl

Project web site: http://www.ict-hd-mpc.eu

Executive Summary

This deliverable report deals with available distributed and hierarchical control schemes applied to a simple toy problem. Once the review is made, a distributed methodology was tested at a benchmark problem. Hence, simulation results are discussed.

Introduction

Hierarchical and Distributed Model Predictive Control (HDMPC) plays an important role in plantwide control, due mainly to its versatility on using the model to optimize the control action by means of a designed cost subject to certain process constraints. On the other hand, large-scale systems deal with complex systems, hard nonlinearities and a large amount of variables. Some examples of large-scale systems are the whole traffic in a city, interconnection of electrical power systems, and complex chemical plants.

Centralized control using MPC has became a powerful tool since 80's due mainly to their use at petrochemical plants. However, as the system gets bigger, the dynamics are more complex, and moreover these dynamics are strongly interconnected, centralized control becomes infeasible, due to computational burden associated with the solution of a bigger optimization problem at each sample time, and the communication bottleneck caused by the information exchange between the centralized CPU and all other entities in the system (sensors, actuators and controllers).

Also, if the system state is not available, a state estimator must be designed in order to solve the optimal control in the MPC sense. Despite the centralized framework has more available theory than the decentralized framework, there are other problems that make this approach unsuitable for large-scale systems: any failure on the data acquisition system, on the system controller, and possibly on the system observer could cause catastrophic consequences on the product, and the plant security. Distributed systems, on the other hand, take into account a system partition in order to solve many optimal subproblems. In this case, coordination mechanisms arise to get an overall plant performance.

While distributed control deals with the coordination and/or cooperation between local agents in a given layer, hierarchical control takes decisions following the transmission of information from higher levels inside a hierarchy. Problems like the guidance of a group of robots, and the scheduling in a plant can be solved by means of hierarchical control.

This paper is organized as follows: first an introduction is made. Then In Chapter 1, a review on hierarchical and distributed MPC schemes applied to large-scale systems are shown in order to give an overview on the state of the art. Chapter 2 deals with a distributed scheme applied to a simple heat conduction and convection problem. Finally some conclusions are given.

Chapter 1

A Review on Distributed Control Schemes applied to simple Toy Problems

Hierarchical and distributed Model Predictive Controllers (MPC) deal with coordinated structures among local controllers to carry out the global plant task in a desired way. Moreover, communication issues due to cooperative strategies, hierarchical coordination, and hard optimization subproblems turn the problem in a difficult task. Hence, toy problems play a crucial role to test the designed methodologies as a previous step before its application in a large-scale system.

In this section, only distributed strategies are applied to this kind of problems and are summarized in order to outline its main features.

1.1 Distributed MPC

In this section, a discussion on the reviewed Distributed Model Predictive Control (DMPC) schemes applied to simple toy problems are presented.

Mercangöz and Doyle III present in [7] a comparison between a fully decentralized MPC and a kind of DMPC design applied to a straightforward four-tank system, as shown in Fig. 1.1. The physical flowsheet of the plant and the mathematical model are used to partition the system into autonomous estimation and control nodes.

In [3] and [8] a distributed control is applied to a system composed by a set of M oscillators with one spatial degree of freedom. These oscillators are coupled by springs connecting each one with the two nearest neighbors. An exogenous vertical force is used as control input for each oscillator as shown in Fig. 1.2

In [2], a system based on pendulums in which each pendulum is connected to another one by a linear frictionless spring is presented. These pendulums are arranged either in straight lines or arrays. The state of each pendulum is composed of its oscillation angle, its rotational angle, and the derivatives of these angles. The controlled inputs are two forces acting horizontally. One force is directed as the horizontal component of the tangent to the trajectory of the pendulums and the other one is orthogonal to the first force. In Figure 1.3, a scheme of the pendulum array is shown.

The control task for the pendulums system is analogous to a transient stability problem in an electric power system in a lower-scale way. Specifically, the pendulums are operated in synchronism mode at a fixed frequency (since the pendulums and springs are frictionless, this condition requires no control forces for its continuation). At time zero, all the pendulums are randomly disturbed. The problem is to bring the pendulums back to their pre-disturbance synchronous condition by simultaneously

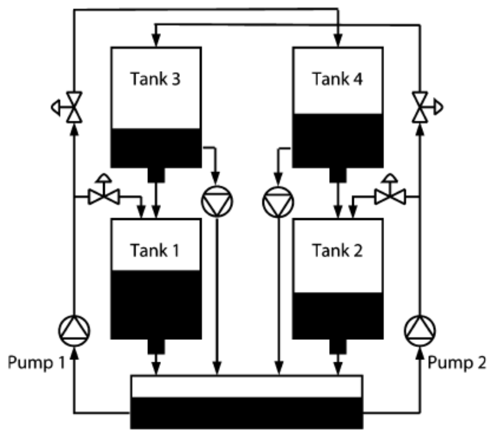

Figure 1.1: Four-tank system. Taken from [7]

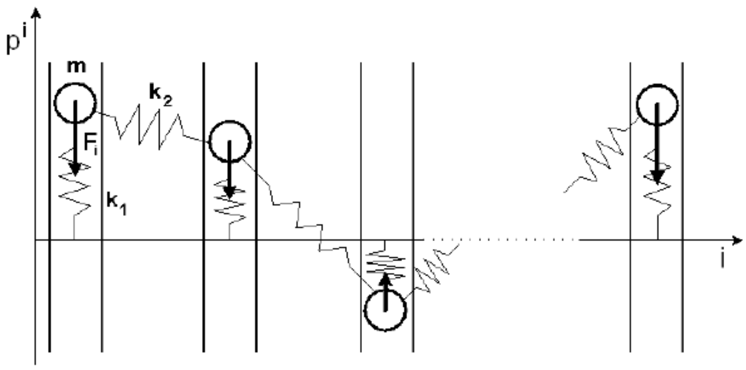

Figure 1.2: Oscillators array. Taken from [3]

minimizing the cumulative error from the desired trajectory and the control-action cost.

Other well-known toy problem to test a distributed model predictive control scheme is the system proposed in [1] and [6]. In these contributions, a two-area electric power system is used to test a distributed model predictive control scheme applied to load frequency control. In a power system with two or more independently controlled areas, the generation within each area must be controlled to maintain the system frequency and the scheduled power exchange. In this example, it was assigned a model predictive controller to control the generator power output directly. Thus, each area can be described as one equivalent generator in series with an impedance. The previous system is also used in [11] to test a distributed model predictive control scheme applied to the automatic generation control stage, adding compensation devices.

In order to illustrate the use of some concepts and approaches of game theory applied to model predictive control, in [9] it is proposed as example a simple vehicle formation. Consider a simple model for the vehicle formation involving three vehicles. Let x_{11}, x_{22} and x_{33} be the positions of the three vehicles relative to some (possibly moving) coordinate system. Assume that the first vehicle controls its own position x_{11} and it is penalized for deviations from a given reference value ν. The second vehicle controls x_{22} and carries a cost depending on x_{22} , but also on x_{11}. Similarly, the third vehicle controls x_{33} and pays a cost that depends on x_{33} and x_{22}. The objective is to find a distributed

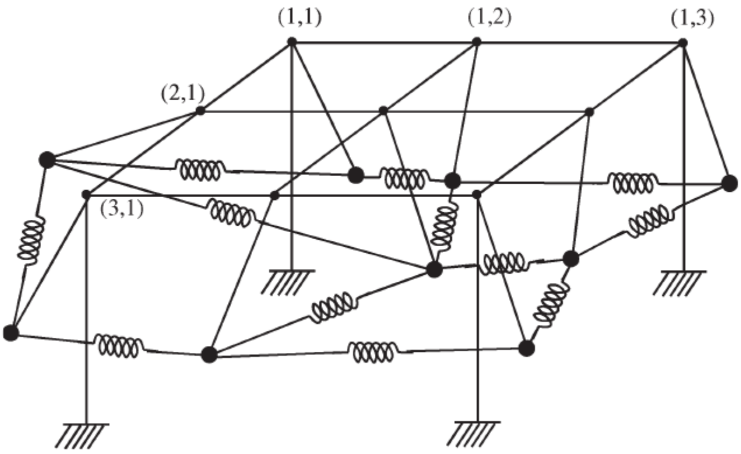

Figure 1.3: Pendulum array. Taken from [2]

scheme for the coordination between vehicles. In Fig. 1.4 the cars array and variables assignation are shown.

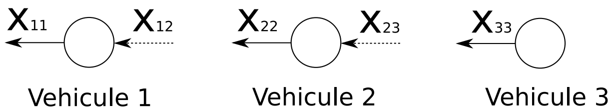

Figure 1.4: Cars array and variables assignation. Taken from [9]

Chapter 2

DMPC applied to a simple toy problem

In order to illustrate the MPC-based methods for distributed control, an easy problem is proposed. This problem is the distribution of an MPC in a spatially distributed system. The concerned phenomena are the heat conduction and convection in a rigid rod.

2.1 A Heat Conduction and Convection model [5]

Consider a solid rod. Assuming a uniform composition of the material, and the heat phenomena are given predominantly at one axis (x-axis), then an energy balance can be made to an internal slice, as it is shown in Fig. 2.1:

$$\rho C_p (A_T \partial x) \frac{\partial T}{\partial t} = \left(\kappa \frac{\partial^2 T}{\partial x^2} + g(x,t) \right) A_T \partial x \tag{2.1}$$

where ρ is the density of the rod, C_p is the heat capacity per unit of mass, κ is the thermal conductivity, P is the perimeter of the cross-sectional circumference, ∂x is the width of the slice, $A_T = P\partial x$ is the area exposed to the environment, T is the temperature inside the slice, x and t are the spatial and temporal variables, and $g(x,t)$ is a generation function defined as:

$$g(x,t) A_T \partial x = \dot{Q}(x,t) P \partial x + h(T_{env} - T(x,t)) P \partial x \tag{2.2}$$

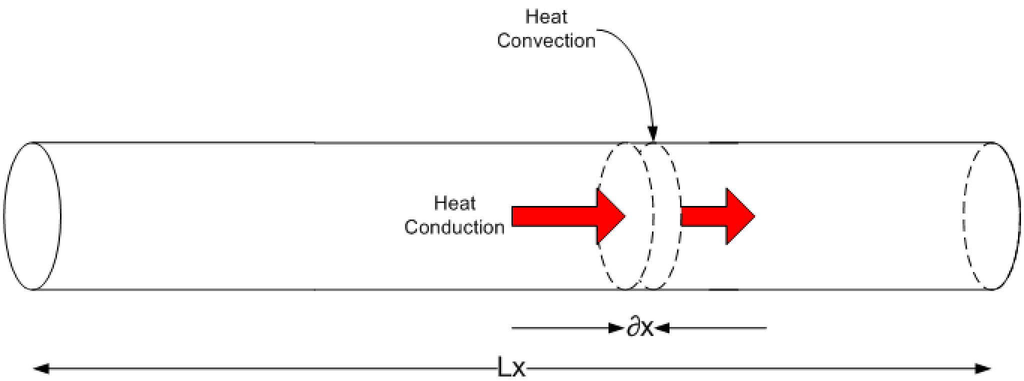

Figure 2.1: A Rod. One dimensional heat exchange.

with $\dot{Q}(x,t)$ the heater power per unit of area, h the convection coefficient, and T_{env} the temperature at the environment.

Then, replacing (2.2) in (2.1), a final PDE is obtained:

$$\frac{\partial T}{\partial t} = \frac{1}{\rho C_p}\left[\kappa \frac{\partial^2 T}{\partial x^2} + \frac{P}{A_T}\dot{Q}(x,t) + \frac{hP}{A_T}(T_{env} - T)\right] \tag{2.3}$$

To model the ends of the rod, an additional assumption must be made: the conduction phenomena is only given in one direction.

2.1.1 Model Discretization

In order to solve numerically the previous model, it is mandatory to approximate the partial derivatives. A straightforward way to do this is applying the finite differences method. This method uses an approximation of the derivatives based on a truncation of the Taylor series at the first order term. Then a derivative can be expressed as:

$$\frac{\partial u}{\partial h} \approx \frac{u_{i+1} - u_i}{\Delta h} \tag{2.4}$$

$$\frac{\partial u}{\partial h} \approx \frac{u_i - u_{i-1}}{\Delta h} \tag{2.5}$$

$$\frac{\partial u}{\partial h} \approx \frac{u_{i+1} - u_{i-1}}{2\Delta h} \tag{2.6}$$

where i is the discretization variable. Eqs. (2.4), (2.5), and (2.6) are known as forward, backward, and central approximations of the derivative respectively. If the previous approximations are successively applied, then second order derivatives can be found as:

$$\frac{\partial^2 u}{\partial h^2} \approx \frac{u_{i+2} - 2u_{i-1} + u_i}{\Delta h^2} \tag{2.7}$$

$$\frac{\partial^2 u}{\partial h^2} \approx \frac{u_i - 2u_{i-1} + u_{i-2}}{\Delta h^2} \tag{2.8}$$

$$\frac{\partial^2 u}{\partial h^2} \approx \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta h^2} \tag{2.9}$$

If the approximations are applied to the rod equation, then three equations must be considered:

$$\frac{dT_i}{dt} = \frac{1}{\rho C_p}\left[\kappa \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2} + \frac{P}{A_T}\dot{Q}_i + \frac{hP}{A_T}(T_{env} - T_i)\right] \tag{2.10}$$

$$\frac{dT_i}{dt} = \frac{1}{\rho C_p}\left[\kappa \frac{T_{i+1} - T_i}{\Delta x} + \frac{P}{A_T}\dot{Q}_i + \frac{hP}{A_T}(T_{env} - T_i)\right] \tag{2.11}$$

$$\frac{dT_i}{dt} = \frac{1}{\rho C_p}\left[\kappa \frac{T_{i-1} - T_i}{\Delta x} + \frac{P}{A_T}\dot{Q}_i + \frac{hP}{A_T}(T_{env} - T_i)\right] \tag{2.12}$$

with i the discretization variable at x-axis, and Δx the length of the differential spatial partition. Eqs. (2.11), and (2.12) are applied at the ends of the rod. Eq. (2.10) is applied inside the rod, when the slice is between two neighboring slices. The number of equations are the same as partitions considered in the rod.

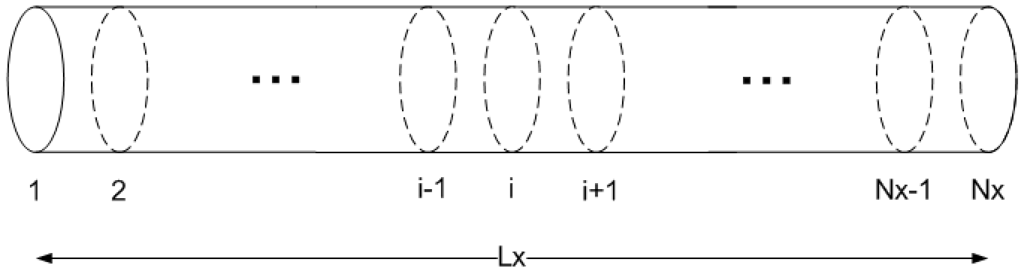

Figure 2.2: Spatial discretization in a rod.

2.1.2 State-Space Model

Consider a generic state-space model:

$$\begin{aligned} \dot{x} &= Ax + Bu_a \\ y &= Cx \end{aligned} \tag{2.13}$$

here x is the state of the system, u_a is the input, and y is the output or measured variables. Moreover $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, and $C \in \mathbb{R}^{n \times p}$, with n is the number of states, m is the number of inputs, and p is the number of outputs. As the new model has finite dimension due to the discretization of the PDEs, a state-space framework can be used to represent the entire physical system.

Consider first a possible discretization of a rod as in the Fig (2.2). The model of the rod can be written as Eq. (2.13), using Eqs. (2.10) to (2.12):

$$A_R = \frac{\kappa}{\rho C_p \Delta x} \begin{bmatrix} -1 & 1 & & & 0 \\ 1 & -2 & 1 & & \\ & & \ddots & & \\ & & 1 & -2 & 1 \\ 0 & & & 1 & -1 \end{bmatrix} - \frac{hP}{\rho C_p A_T} \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} \tag{2.14}$$

$$B_R = \left[\begin{array}{ccccc|c} \frac{P}{\rho C_p A_T} & 0 & \cdots & & 0 & \frac{hP}{\rho C_p A_T} \\ 0 & \frac{P}{\rho C_p A_T} & & & \vdots & \frac{hP}{\rho C_p A_T} \\ \vdots & & \ddots & & & \vdots \\ & & & \frac{P}{\rho C_p A_T} & 0 & \frac{hP}{\rho C_p A_T} \\ 0 & \cdots & & 0 & \frac{P}{\rho C_p A_T} & \frac{hP}{\rho C_p A_T} \end{array} \right] \tag{2.15}$$

with $x = \begin{bmatrix} T_1 & T_2 & \cdots & T_{N_x-1} & T_{N_x} \end{bmatrix}^T$, $u_a = \begin{bmatrix} q_1 & q_2 & \cdots & q_{N_x-1} & q_{N_x} & T_{env} \end{bmatrix}$, $q_1, q_2, \ldots, q_{N_x-1}$, and q_{N_x} the heat power applied to each resistance or actuator, A_R, and B_R the rod matrices. C_c depends on the number and location of measured states.

2.1.3 Model parameters

As it was mentioned, the model parameters are strongly dependent of the material of any element. In this case, the model parameters were taken from [10], and these are consistent with EN AW1350-F Aluminum. Also, it is always assumed that the rod is inside a room with a given environment temperature. The parameters are presented in Table 2.1. Further details of the parameters can be found in [10].

Table 2.1: Model parameters

Symbol	Parameter	Min	Nominal	Max	Units
ρ	Density	2600	2700	2800	$Kg.m^{-3}$
κ	Thermal conductivity	230	230	234	$W.m^{-1}.K^{-1}$
C_p	Heat capacity per mass unit	900	900	900	$J.K^{-1}.Kg^{-1}$
$\dot{Q}_{R\max}$	Max heat power per area unit (rod)	0	200	2000	$W.m^{-2}$
h	Convection coefficient	2	10	25	$W.m^{-2}.K^{-1}$
T_{env}	Environment temperature	291	298	303	K

2.2 Centralized MPC (CMPC) as a reference controller

In this Section, a centralized MPC is shown in order to get a baseline for the performance such that it can be compared with the DMPC scheme. Consider a rod with a length of $2m$, and it is discretized in space such that there are 20 partitions. Then a state-space model composed by 20 differential equations can be obtained as it was presented earlier. In this case, 5 manipulated variables were assumed, that is, 5 heaters located at 1, 5, 10, 15, and 20 discrete points into the rod. With this configuration, state controllability can be proven at the system. For control purposes, it is assumed total accessibility of the system state by means of an ideal observer.

A linear MPC controller is designed with a prediction horizon of 10 time steps and a control horizon of 2 time steps. The control actions are constrained following the parameters afore mentioned. Consider first a regulation problem with a set point of $305K$. Figure 2.3 shows the closed loop response of the system.

From figures 2.3 to 2.6, it can be seen that the controller regulates the system states at the desired values at the heating points, but in the remaining points, due to the convection with the environment and with the neighboring points, the temperature profile falls, leading a steady-state error (Figures 2.3 and 2.4). Moreover, the control inputs applied by the CMPC are inside the domain defined for their ($0 \leqslant u \leqslant 2000W$). It can also be seen an oscillatory behavior at the beginning of the simulation, until the steady state of the system is achieved.

2.3 DMPC Controller

To implement the centralized MPC, the following cost function was used:

$$\min_{u(k)} \sum_{t=1}^{N_p} [e^T(k+t)Qe(k+t)] + \sum_{t=1}^{N_u} [u^T(k+t)Ru(k+t)] \tag{2.16}$$

where $N_p \geq N_u, \in \mathbb{R}$, and $e(k+t) = y_{ref}(k+t+1) - y(k+t+1)$.

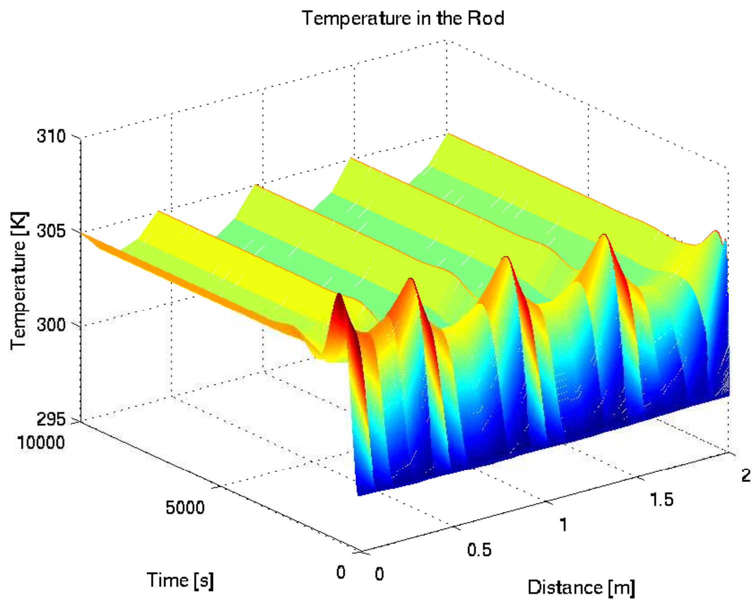

Figure 2.3: Dynamical response of the controlled system

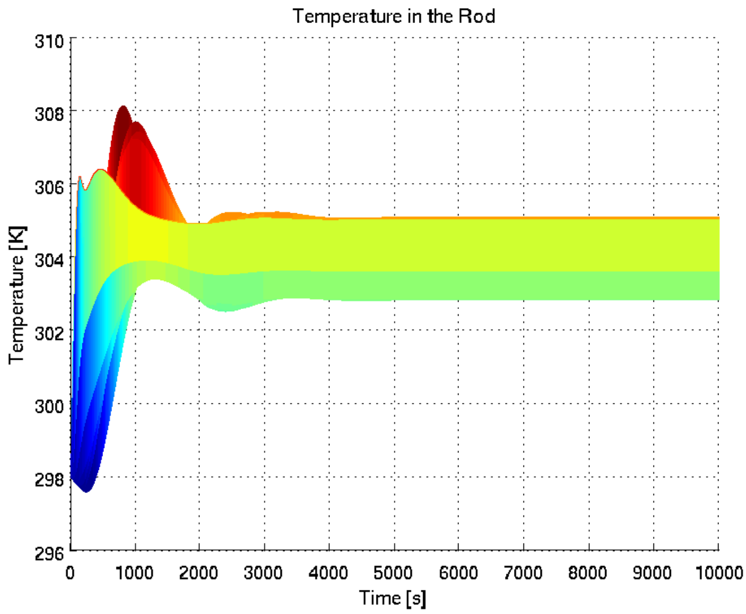

Figure 2.4: Dynamical response of the controlled system (Temperature vs time plane)

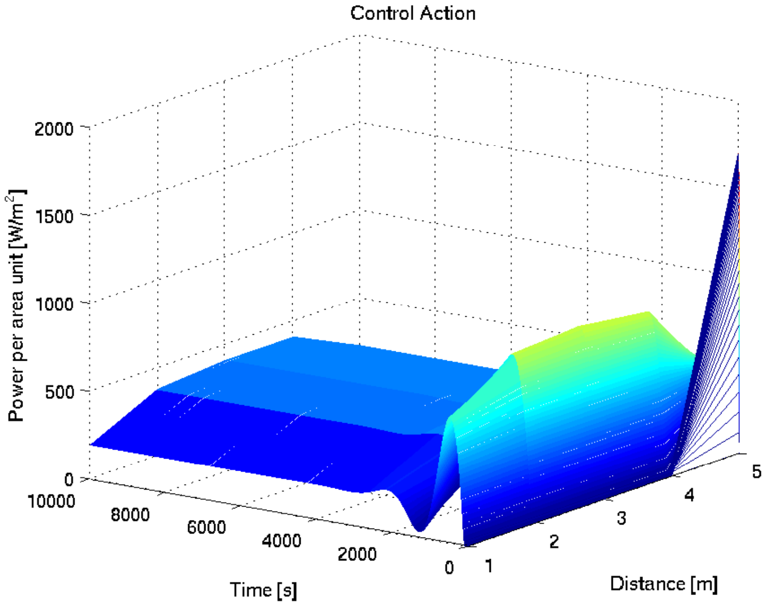

Figure 2.5: Manipulated variables

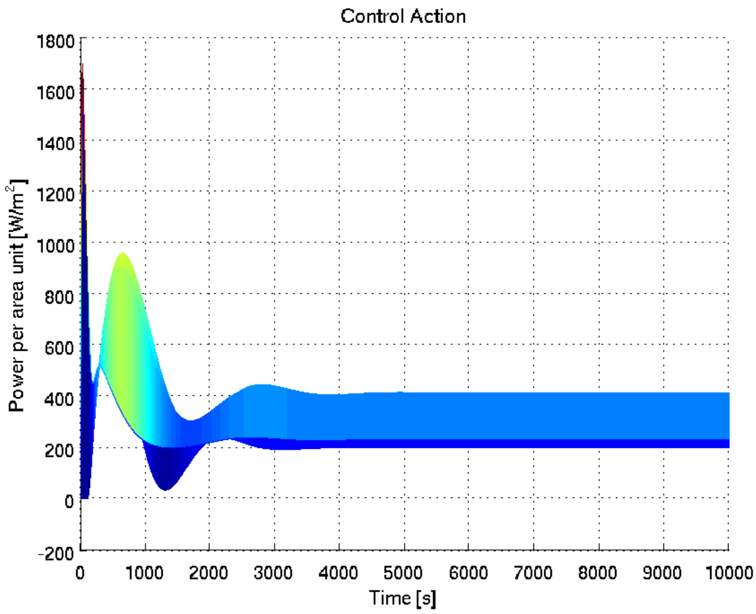

Figure 2.6: Manipulated variables (applied power vs time plane)

From the discrete linear model of the system

$$\begin{aligned} x(k+1) &= A_{dR}x(k) + B_{dR}u(k) \\ y(k+1) &= C_{dR}x(k+1) \end{aligned} \tag{2.17}$$

being A_{dR}, B_{dR}, C_{dR} the matrices associated with the discrete model of the system, it yields

$$e(k+t) = y_{ref}(k+t+1) - C_{dR}[A_{dR}x(k+t) + B_{dR}u(k+t)] \tag{2.18}$$

Since the discretization process does not change the diagonal structure of the matrices A_R, B_R, C_R of the continuous model of the system, the rod model can be decomposed into several subsystems coupled by its dynamics. In this case, only two subsystems are considered. Thus the discrete model of the plant can be expressed as

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \\ \begin{bmatrix} y_1(k+1) \\ y_2(k+1) \end{bmatrix} &= \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} \end{aligned} \tag{2.19}$$

where $A_{ij}, B_{ij},\ i \neq j$ are the matrices representing the interaction between the subsystems.

From the block formulation 2.19, the prediction error can be written as

$$e(k+t) = \begin{bmatrix} y_{1ref}(k+t+1) - C_1x_1(k+t+1) \\ y_{2ref}(k+t+1) - C_2x_2(k+t+1) \end{bmatrix} \tag{2.20}$$

Thus the quadratic term associated with the error in the MPC formulation becomes

$$e^T(k+t)Qe(k+t) = \begin{bmatrix} y_{1ref}(k+t+1) - C_1x_1(k+t+1) \\ y_{2ref}(k+t+1) - C_2x_2(k+t+1) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} y_{1ref}(k+t+1) - C_1x_1(k+t+1) \\ y_{2ref}(k+t+1) - C_2x_2(k+t+1) \end{bmatrix} \tag{2.21}$$

Assuming $Q_{ij} = 0\ \forall\ i \neq j$ the expression 2.21 is equals to:

$$\begin{aligned} e^T(k+t)Qe(k+t) &= e_1^T(k+t)Q_{11}e_1(k+t) + e_2^T(k+t)Q_{22}e_2(k+t) \\ &= \sum_{i=1}^{2} e_i^T(k+t)Q_{ii}e_i(k+t) \end{aligned} \tag{2.22}$$

Therefore, the global cost function can be expressed as the sum of several local cost functions (in this case two), a DMPC can be applied to control this system and its formulation can be written as:

$$\begin{aligned} &\min_{u_i(k)} \sum_{i=1}^{n} J_i[x_i(k,t), u_i(k,t)] \\ s.t: x_i(k+t+1) &= A_ix_i(k+t) + B_iu_i(k+t) \\ y_i(k+t+1) &= C_ix_i(k+t) + D_iu_i(k+t) \\ &\text{operational constraints} \end{aligned} \tag{2.23}$$

where $J_i[x_i(k,t), u_i(k,t)] = \sum_{t=1}^{N_{pi}}[e_i^T(k+t)Q_{ii}e_i(k+t)] + \sum_{t=1}^{N_{ui}}[u_i^T(k+t)R_{ii}u_i(k+t)]$, $Q_{ii}, R_{ii} > 0$.

Now, in order to test the DMPC formulation for the Heat Conduction and Convection System, two situations are proposed:

1. Case 1:To regulate all temperatures of the rod at 350K
2. Case 2:To regulate all temperatures of the rod at 350K, except in the subsystems boundary where the reference value was 400K

The performance of the system in the first case is shown in the Figure 2.7. The DMPC proposed tries to control the temperature profile of the rod (Figure 2.7) as it can be seen. However due to the prediction model has a big error at the point where the subsystems are coupled, the set point achievement is not possible as with the CMPC.

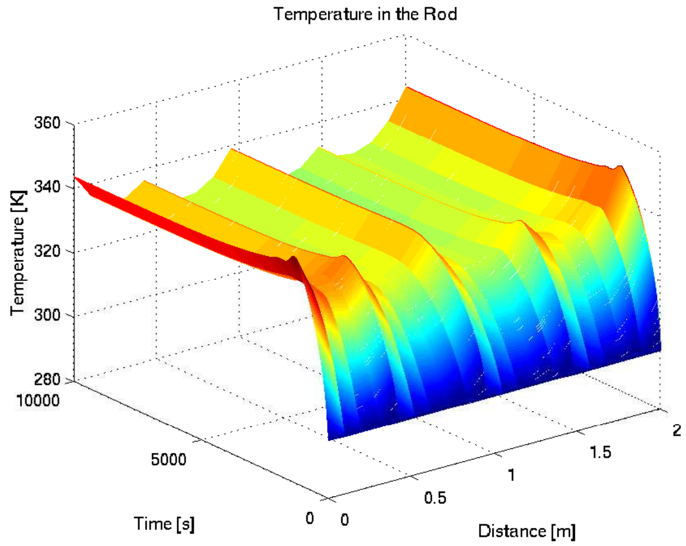

Figure 2.7: Temperature profile along the rod (first case)

Figures 2.8 and 2.9 present the behavior of the system in the second case. Figure 2.8, shows the temperature profile of the rod. Similar to the first case, the performance achieved by the DMPC is worst than the achieved with the centralized MPC, in terms of the deviation from the reference value. However, due to the reference value in the common boundary of the subsystems was different than the other points along the rod, the performance in this case is better than the achieved in the first case. The simulated temperature profile for the rod in this case is shown in Figure 2.9.

The control actions are shown in Figures 2.10 to 2.12. Note that the constraints of the control inputs are satisfied, that is, ($u_i \leqslant 2000W$).

Comparing the control actions of both model predictive control schemes (CMPC and DMPC), it is possible to see that the control actions applied by the distributed schemes are smoother than the ones applied by the centralized controller. Also, in the distributed scheme the control actions do not exhibit the oscillatory behavior presented by the control actions of the centralized controller.

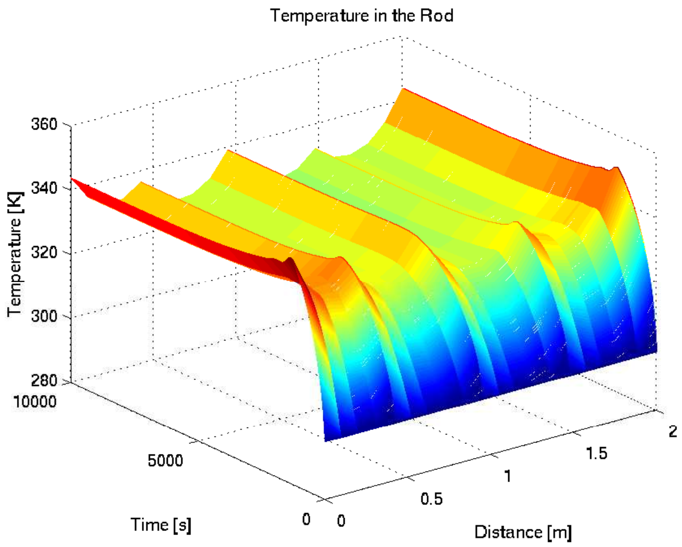

Figure 2.8: Temperature profile along the rod (second case)

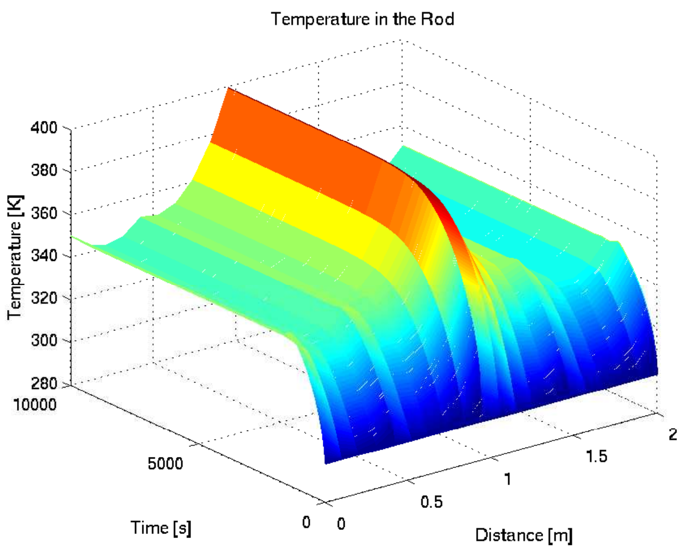

Figure 2.9: Predicted temperature profile along the rod (second case)

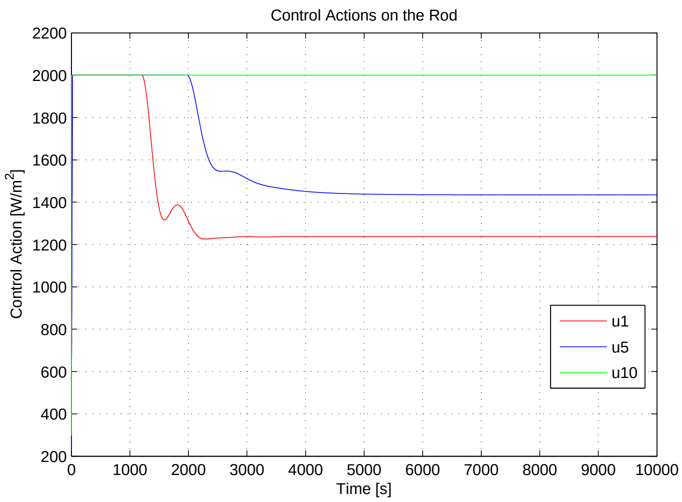

Figure 2.10: Control actions of the subsystem one (second case)

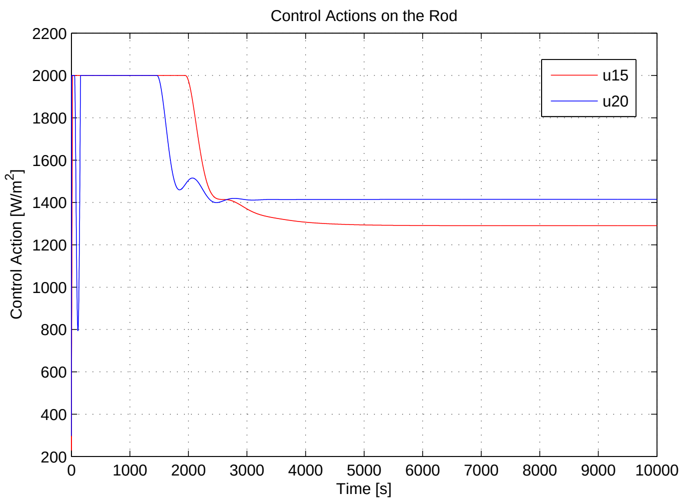

Figure 2.11: Control actions of the subsystem two (second case)

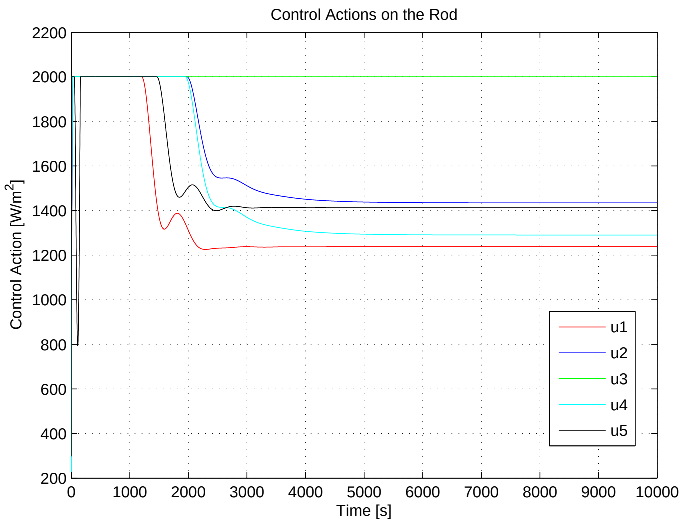

Figure 2.12: Control actions of the whole system (second case)

Chapter 3

Conclusions

The reviewed distributed scheme makes clear the fact of having strategies dependent on the interaction among subsystems (interactive dynamics, interactions among local cost functions, and interaction among local constraints from one node to each other). Then, solutions are based on those types of interaction, making the theory of distributed control hard to generalize. Due to these difficulties, toy problems play an important role in order to identify the possible troubles in large-scale systems implementations.

A distributed model predictive control strategy was tested, based on the fact that the performance cost function can be decomposed as a sum of several local functions (in this case two). In this strategy a one step time delay was considered in the communication process. This strategy was tested in a heat convection and conduction system. The performance of the system with the proposed strategy was compared with the performance achieved using a centralized model predictive control scheme. From this comparison, it is possible to conclude that it is imperative to improve the data reconciliation at the common boundary of the subsystems, in order to achieve a better performance in DMPC schemes, applied to systems continuous in space.

Finally it can be stated that toy problems allows to understand some troubles of large-scale systems like coupling, interconnection and interaction among subsystems. These issues are useful mainly to design of MPC strategies able to guarantee the handling of these additional phenomena.

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