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Executive Summary

Important literature of the 1960s and 1970s on hierarchical and distributed model predictive control (HD-MPC) is reviewed. Main properties of the existing methods are stated. Though, a comparison is quite difficult, as methods of that time period are more conceptual methods than applicable methods. This report tries the descibe the relationships between the different methods. It focuses on literature, which has been neglected in recent research. Then, present literature is reviewed, in order to get a summary of today's research in the field of hierarchical and distributed control methods. Present methods are quite heterogeneous with repsect to the considered class of systems, thus an objective and comprehensive analysis is almost impossible. Hence this report is giving an overview on the current focus in research. Focus of present research is mainly on linear methods and there is a clear lack for nonlinear methods. Hence, there has to be a shift in research from linear to nonlinear HD-MPC methods. Furthermore, it seems to be interesting to look back on some ideas of the first HD-MPC literature of the 1960s and 1970s for the upcoming research.

Chapter 1

Introduction

Hierarchical and distributed model predictive control methods have quite a long history. First hierarchical and distributed control structures have been introduced for linear frequency based control methods: On the one hand cascade control structures as depicted in Figure 1.1 and on the other hand dectralized SISO Controllers for a MIMO type of process as depicted in Figure 1.2. Both methods still belong to the most important control structures in industry, as the methods offer some nice properties: For instance they can easily be implemented, they require only little computational power, they offer easy maintainability, and the methods are well understood.

However, there are some limits to these control methods. The most important one might be the fact, that these control methods are not optimal control methods. Though, as optimality, either economically or ecologically, is of increasing importance today, hierarchical and distibuted optimization based control methods, namely model predictive control methods get into the focus of research. Another limit of the classical control methods is the fact, that interactions within spatially distributed systems as within the MIMO process P in Figure 1.2 are either neglected or assumed to be disturbances [1, 2, 3, 4]. As a result stability and robustness of the control structure might be decreased [5, 4]. Even if these properties are not jeopardized, and if the decentralized controllers C_1 and C_2 are optimal controllers, the neglection of interactions will result in a global control structure, that is not necessarily optimal [6, 7]. Furthermore, many of today's control methods do not feature the explicit consideration of constraints on the input variables or on the state variables. Other methods only consider linear systems, such as $\mathscr{H}_\infty$-control [8].

However, centralized model predictive control features all of the above-mentioned properties: Optimality, guaranteed stability for MIMO processes, consideration of input and path contraints, and applicability to nonlinear systems. But there are some disadvantages of centralized model predictive

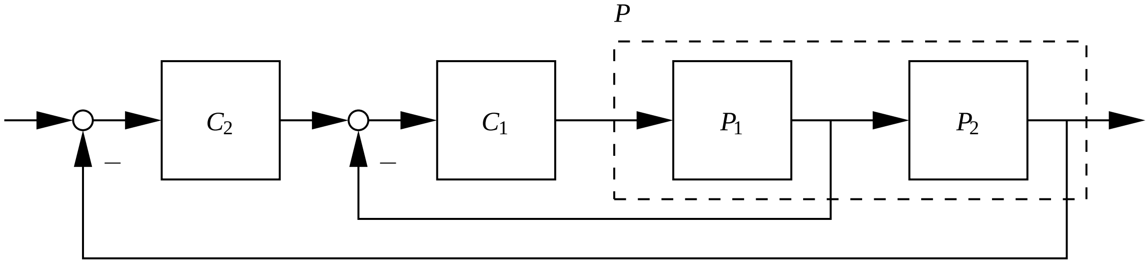

Figure 1.1: Classical hierarchical cascade control structure: Control is separated into a low level controller C_1 and a high level conrtoller C_2.

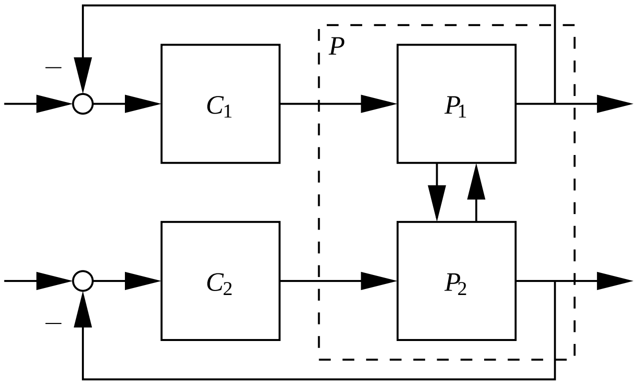

Figure 1.2: Classical decentralized control structure: Multiple SISO controllers C_i control a MIMO process P.

control methods:

- If the size of the considered processes increases or the process time constants decrease, centralized model predictive control will demand tremendous computing power and thus will not be appplicable for a real-time application [6, 4];
- in case of a spatially distributed system, e.g. a water supply network, communication among different subsystems of the global process might be limited;
- maintainability of a centralized large-scale control structure is difficult [9];
- and reliability of a decentralized control structure might be better than the one of a centralized control structure [6].

Hence, hierarchical and distributed model predictive control methods are assumed to be the methods combining the advantages of centralized and decentralized model predictive control, while solving their disadvantages.

In the following some nomenclature is introduced. On the one hand we consider a hierarchical spatially distributed control structure as depicted in Figure 1.3 [10]. The MIMO process P is considered of beeing composed of multiple interacting subprocesses P_i. The control structure consists of multiple local controllers $C_i,\ i = 1, \dots, N$ and a supervisory controller C_0. The local controllers, also called infimal [10] controllers, only interact with the corresponding subprocesses and the supervisory controller. The latter one, which is also called coordinator [11] or supremal [10] controller, only interacts with the local controllers. This hierarchical control structure is also referred to as coordinated control [11].

On the other hand distributed control structures with only a single control layer can be considered, as depicted in Figure 1.4 [10]. While in case of the hierarchical control structure, communication between local controllers is achieved by coordination of the supervisor, in case of the distributed control, which is also called cooperative control [11], the supervisor is completely missing. Thus, all local controllers have to communicate directly with each other. The hierarchical and distributed control structures can be furtherly classified, e.g. by the type of communication protocol: A wide overview is presented in [11].

These two control structures possess different advantages and disadvantages: As the distributed structure does not rely on the supervisor, this structure seems to have better reliability properties,

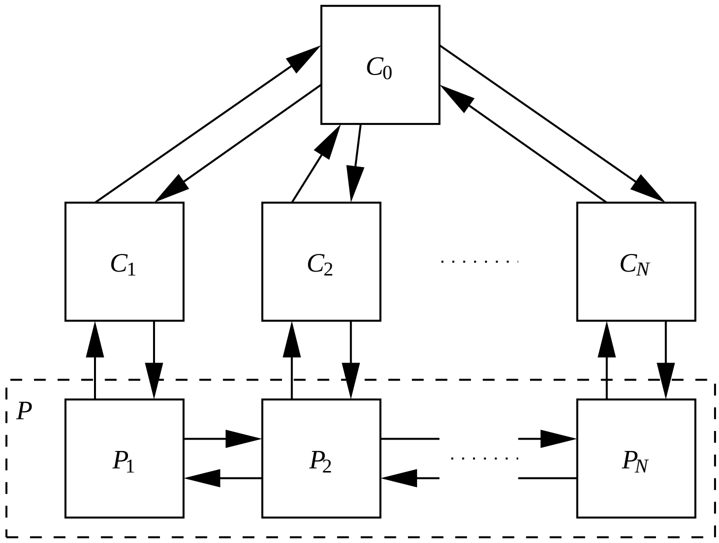

Figure 1.3: Hierarchical spatially distributed control structure: A MIMO process P is considered of beeing composed of multiple interacting subprocesses P_i. The control structure consists of multiple local controllers $C_i,\ i = 1,\ldots,N$ and a supervisory controller C_0.

while the hierarchical structure will fail, in case of a breakdown of the supervisor. On the other hand, the topological structure will be easier for the hierarchical, as the number of subsystems and local controllers N increases: While in case of the hierarchical structure it is necessary the implement N bidirectional links between the supervisor and the local controllers, in case of one-layer control structure it will be necessary to implement $N \cdot (N-1)/2 \sim N^2$ bidirectional links, if each local controller needs to communicate to all other local controllers. Hence, for processes consisting of many subsystems, it appears to be reasonable to have a hierarchical control structure at least in order to simplify the communication among different local controllers.

In model-predictive control, even for centralized MPC structures, it is quite common already today to have a hierarchy within the control structure. That hierarchy, however, deals with the different time

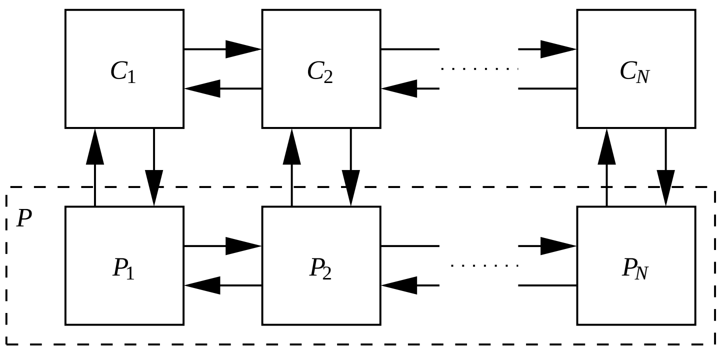

Figure 1.4: One-layer distributed control structure: A MIMO process P is considered of beeing composed of multiple interacting subprocesses P_i. The control structure consists of multiple local cooperating controllers $C_i,\ i = 1,\ldots,N$.

scales [12, 13] of the controlled processes and is also referred to as vertical decomposition [14, 15, 16]. This vertical decomposition has to be distinguished from the horizontal or spatially decomposed problems, that we like to consider in hierarchical and distributed MPC. Usually, there are at least two control layers in a vertically decomposed system: A linear or nonlinear MPC on the lower layer and a static real time optimization or set point optimization on the upper control layer [11, 6]. Other topologies in vertical decomposition additionally include PID-based control on the lowest level [11, 12]. Another variation of that topology replaces the static real time optimization layer by a dynamic real time optimization (D-RTO) layer [16, 17]. Alternatively, there may be multiple MPC layers, that take into account different time-scales of the controlled processes [13]. However, recently there is also some effort in order to reduce the number of different control layers and replace them by a single economic nonlinear control layer [18].

The basis for all of these layers in model predictive control is the optimization of static or dyamic systems, i.e. the minimization of an objective function subject to the system equations, its initial values, its input constraints, the constraints on the state variables, and some endpoint constraints. Thus, in the following section the mathematical problem for hierarchical and distributed model predictive control is formulated. In order to be more general, we stick to the dynamic nonlinear formulation of the optimization problem.

Chapter 2

Mathematical problem formulation

The idea of model predictive control is to numerically solve a dynamic optimization problem at each sampling time. Though only the control signals for the first subsequent period of time are applied to the process. Then the optimization problem has to be solved again for new initial conditions and again also only the first values for the manipulated variables are applied to the process. This method is repeated for each of the following sampling times of the control problem.

The same idea is to be implemented in hierarchical and distributed MPC. Thus, the basis for HD-MPC is an optimization problem, that has to be decomposed for different subsystems. As various versions of MPC are considered, e.g. linear and nonlinear MPC, discete-time and continuous-time MPC, there exist also many different formulations of the optimization problem. Exemplarily in the following a quite general mathematical formulation of the control problem is introduced. We consider the global nonlinear optimal control problem:

$$\min_u \Phi(t,x,u,m), \tag{2.1a}$$

$$\text{w.r.t.} \quad \dot{x} = f(t,x,u,m), \quad x(0) = x_0, \tag{2.1b}$$

$$0 \leq g(t,x,u,m), \tag{2.1c}$$

$$0 \leq \pi(t_f, x(t_f), u(t_f), m(t_f)), \tag{2.1d}$$

$$m = H\,[x^T, u^T]^T, \tag{2.1e}$$

where $\Phi \in \mathbb{R}$ is the objective function, $t \in \mathbb{R}$ is the time, $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^p$ is the input vector, $m \in \mathbb{R}^q$ is the the vector of coupling or interaction variables, i.e. the variables, that represent the couplings between different subsystems, and t_f is the final time of the optimization problem. The system dynamics are represented by equation (2.1b), the input and state constraints are summarized in equation (2.1c), and equation (2.1d) contains the constraints for the final time. The last equation (2.1e) describes the couplings within the system and are called coupling constraints.

In case of the hierarchical and distributed model predictive control problem, the global optimization problem (2.1) has to be solved by the solution process of local problems for the N subsystems, which are described by the following local optimization problems:

$$\min_{u_i} \Phi_i(t, x_i, u_i, m_i), \tag{2.2a}$$

$$\text{w.r.t.} \quad \dot{x}_i = f_i(t, x_i, u_i, m_i), \quad x_i(0) = x_{i,0} \tag{2.2b}$$

$$0 \le g_i(t, x_i, u_i, m_i) \tag{2.2c}$$

$$0 \le \pi_i(t_f, x_i(t_f), u_i(t_f), m_i(t_f)), \tag{2.2d}$$

$$m_i = H_i\,[x^T, u^T]^T. \tag{2.2e}$$

Thereby x_i is the local state vector, u_i is the local input vector and m_i are the local coupling variables of the subsystem i. The local interaction variables depend on the global state and input variables. Hence, in case of hierarchical and distributed model predictive control one task of coordination or cooperation is to provide meaningful information on how to derive the local interaction variables.
Keep in mind, that the objective function is not necessarily additive, i.e. in general

$$\Phi(t, x, u, m) \neq \sum_{i=1}^{N} \Phi_i(t, x_i, u_i, m_i)\ . \tag{2.3}$$

Furthermore, the set of state variables $\{x_i | i = 1, \dots, N\}$ does not necessarily have to be a disjunct set. However, usually the objective function Φ is assumed to be additive and the set of state variables is assumed to be disjunct.

The second task of coordination and cooperation is to modify the local optimization problems (2.2), such that the accumulated result $\{u_i^{\text{opt}} | i = 1, \dots, N\}$ of the local optimization problems (2.2) is the same as the result u^{opt} of the global optimization problem:

$$u^{\text{opt}} \stackrel{!}{=} \left[u_1^{\text{opt}}, u_2^{\text{opt}}, \dots, u_N^{\text{opt}}\right]^T \tag{2.4}$$

For this purpose, the local optimization problems (2.2) can be adapted in different ways: Different ideas are stated in the following section.

Chapter 3

Origins of hierarchical and distributed MPC methods

Development of spatially distibuted optimization methods has started already in the 1960s. As there exist already many literature reviews in the field of hierarchical and distributed model predictive control and large-scale systems, which cover the literature of the 1960s and 1970s, we will first refer to these review papers and then stress some important literature of that period of time.

3.1 Introductory and survey literature

In 1973, Smith et al. published an introductory overview on the topic of hierarchical systems theory [19]. Singh et al. concentrated on available control methods for interacting dynamical systems, which can practically be applied [20]. In 1977, Mahmoud presented a very comprehensive overview [21], which is considered to the most important survey paper of the 1970s. The paper covers a wide range of the research progress in literature. While the main subject of that article covers multilevel, i.e. hierarchical, optimization techniques, it also covers the early progress in multilevel systems identification as well as the application to water resource systems. Mahmoud emphasizes the difference of fesible and infeasible methods [22]. The difference is especially important in real-time applications, when full convergence of the methods migtht not be possible. In 1978, Sandell et al. presented another survey [23]. This survey covered the topics of model simplifications, stability analysis of interconnected systems and decentralized control methods.

3.2 Fundamental literature in early HD-MPC research

In 1960, Dantzig and Wolfe proposed a decomposition method for linear programs [24], which is often referred to as *Dantzig-Wolfe decomposition*. Dantzig and Wolfe consider a large-scale linear programming problem, i.e. a linear objective with respect to linear constraints. The problem is decomposed into subproblems, which can be solved independently. These subproblems are coordinated by a master problem [6]; thus the method belongs to the hierarchical methods.

Presumably the initially most important progress has been achieved and summarized in a monograph by Mesarovic et al. [10]. A summary of the main concepts of Mesarovic can also be found in

[21]. Mesarovic et al. formulated some very general principles on how a coordinated control structure can be implemented, namely the *Interaction Balance Principle* and the *Interaction Prediction Principle* [25].

The Interaction Balance Principle considers the interaction variables m_i as degrees of freedom for the local controllers C_i, which compute desired interaction variables $\hat{m}_i$. The task of the coordinator C_0 is to calculate coordinating signals α_i, based on the errors of desired and real interaction variables $\varepsilon_i = \hat{m}_i - m_i$. The overall optimum is achieved if the actual interaction inputs m_i are precisely those, which are calculated by the local optimizations, i.e. $\hat{m}_i = m_i, \ \forall\ i \in [1, \ldots, N]$ [25]. An illustration of that principle is depicted in Figure 3.1 for the case of a decomposition of the process P into two subsystems P_1 and P_2.

On the other hand within the Interaction Prediction Principle only the local input variables u_i are considered as degrees of freedom for the local controllers C_i. The task of the coordinator C_0 is to predict interaction inputs β_i. The overall optimum will then be achieved, if the predicted interaction inputs β_i are correct, i.e. $\beta_i = m_i, \ \forall\ i \in [1, \ldots, N]$ [25]. The control topology of the Interaction Prediction Principle is depicted in Figure 3.2 for a system P, which is decomposed into two subsystems.

The very general theory of the Interaction Balance and Prediction Principles is augmented by further important properties of decomposed control structures [10]. The notion of *coordinability* is defined and necessary as well as sufficient conditions for this property are given. Coordinability can be defined as follows [21]: The examined system is coordinable by a coordination principle if the principle is applicaple and if there exists a coordination parameter such that the corresponding coordination condition is satisfied. Hence, this is a convergence condition, whether the method of the decomposed problem will converge to the optimal solution of the centralized problem or not.

Coordination can be achieved by the intervention of the subsystems using coordination parameters, which can be divided into two subsets [21]: On the one hand the submodels can be modified, which is called model coordination, on the other hand in goal coordination the objective function of the local problems are modified. Other important properties, which are defined, are the *additivity* (see Chapter 2) of objective functions and *harmony* of different objective functions ϕ and $\tilde{\phi}$: The objective function ϕ is in harmony with another objcetive function $\tilde{\phi}$, if the optimal solution u^{opt} for the minimization problem of ϕ is also the the vector that minimizes $\tilde{\phi}$, i.e. $u^{\text{opt}} = \tilde{u}^{\text{opt}}$. However, the values of the objective functions do not necessarily have to match.

Moreover, Mesarovic et al. introduce some new operators, the so-called *goal-interaction operators* [10], which can be applied to modify local objective functions. These operators descibe the influence of input variables on the objective function of the global control problem:

- The *Total Goal-Interaction Operator* [10] gives a measure of the change in the overall cost/objective function, which is causes by the change of a local control input through nonlocal variables.
- The *Partial Goal-Interaction Operator* [10] is also a measure of the change in the overall cost/objective function, which is caused by the change of a local control input through a specific interaction input.
- The *Interface Goal-Interaction Operator* [10] describes the change of the overall cost/objective function, which is caused by a change of a interaction input.

Surprisingly, the theory of interaction operators has hardly or even not all been persued in subsequent research.

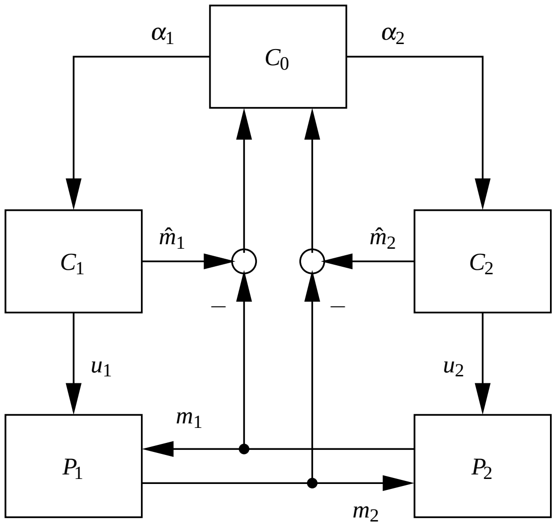

Figure 3.1: Interaction Balance Principle for two subsystems [25].

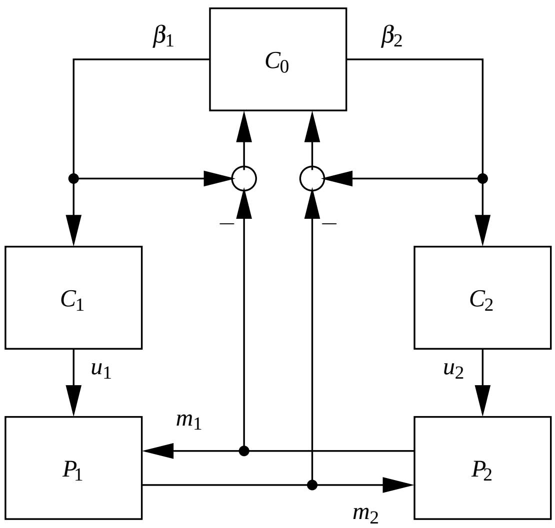

Figure 3.2: Interaction Prediction Principle for two subsystems [25].

Another important monograph of the 1970s has been presented by Lasdon [26]. The monograph starts at basics of linear as well as of nonlinear optimization theory. The main part discusses methods for the optimization of large scale systems with some focus on decomposition methods, e.g. the Dantzig-Wolfe decomposition [24] is reviewed. Although Lasdon discussed many other methods in detail, the one, which has been in the interest of subsequent research and which has later been denoted as one of the most promising methods for decentralized optimization [20] is the dual optimization method [27]. The basic idea of dual optimization will be descibed in the following paragraph.

In dual optimization, additionally to the primal optimization problem (2.1) also its dual problem is considered. We assume that the objective function is additivie. Then, the dual problem is

$$\max_{\lambda} \varphi(t,x,z,u,m,\lambda) \tag{3.1a}$$

$$\text{s.t. } \varphi(t,x,z,u,m,\lambda) = \min_{x,u} L(t,x,z,u,m,\lambda) = \min_{x_i,u_i} \sum_{i=1}^{N} L_i(t,x_i,z_i,u_i,m_i,\lambda) \tag{3.1b}$$

with respect to

$$L_i(t,x_i,z_i,u_i,m_i,\lambda) = \Phi_i(t,x_i,u_i,m_i) + \lambda^T \cdot h_i(x_i,u_i,m_i), \tag{3.2a}$$

$$\dot{x}_i = f_i(t,x_i,u_i,m_i), \quad x_i(0) = x_{i,0} \tag{3.2b}$$

$$0 \leq g_i(t,x_i,u_i,m_i) \tag{3.2c}$$

$$0 \leq \pi_i(t_f, x_i(t_f), u_i(t_f), m_i(t_f)), \tag{3.2d}$$

$$h_i(x_i,u_i,m_i) = m_i - H_i\,[x^T, u^T]^T\ . \tag{3.2e}$$

Here we consider an equality form of the interacion constraints (3.2e) as described in [28]. The set of new local optimization problems

$$\min_{x_i,u_i} L_i(t,x_i,z_i,u_i,m_i,\lambda) \tag{3.3a}$$

$$\text{w.r.t (3.2)} \tag{3.3b}$$

can then be solved independently. In order to achieve a solution of the global problem, the dual problem (3.1) has to be solved, e.g. by a coordinator. The resulting updated Lagrange multipliers λ, which can be interpreted as prices, are used to coordinate the infimal problems (3.3). Hence dual-optimization belongs to the class of price-driven coordination methods. It is a realization of the Interaction Balance Principle and, as the objective function is modified, belongs to the goal-coordination methods. A clear disadvantage of the method is the fact, that the method is infeasible and thus requires full convergence to achieve a feasible solution. Here, infeasibility means, that the interconnection constraints (3.2e) are only fulfilled after full convergence of the method.

In 1980, Findeisen et al. presented a monograph [28], that is based on many of the methods which have been presented in the books of Mesarovic et al. and Lasdon: e.g. there are extensions to constrained and dynamic optimization. A short review can be found in [29].

Chapter 4

Present focus in research

There is also some recent summarizing literature on hierarchical and distributed model-predictive control, e.g. Rawlings and Stewart [30] discuss oppurtunities and challenges of research in HD-MPC, e.g. the robustness to model errors or how to deal with communication disruption. Scattolini [11] focuses on different hierarchical and distributed control topologies and also discusses the states open questions for the future, such as how to partition the system.

In the following sections the present focus in research for hierarchical and distributed control systems is summarized. On the one hand, methods for multi agent systems are summarized. On the other hand, the main focus is on methods for systems in process engineering.

4.1 Distributed control for multi agent systems

Recent research in the field of distributed and hierarchical control is mainly driven by applications such as unmanned and autonomous vehicles [31, 32, 33], and in particular wireless communication. Typically these systems are modelled by linear dynamics as kinematic agents with $\dot{x}_i = u_i,\ x_i \in \mathbb{R}^\nu$, as dynamic agents with $\ddot{x}_i = u_i,\ x_i \in \mathbb{R}^\nu$ or sometimes by dynamics of even higher order [34]. Thereby ν is the dimension of the considered space which is typically $\nu = 2$ or $\nu = 3$. An important property of this class of systems, commonly referred to as Multi Agent Systems (MAS), is that the different subsystems, the agents, are origininally not coupled. But there are some common goals such as consensus [34], pattern formation, flocking or the avoidance of collisions [31, 35, 36]. In order to achieve these common goals coupling control methods are introduced. Even though MPC is an important control method in the field of MAS, research is not limited to optimization based control methods. Other modern control tools applied, are for instance linear matrix inequalities [33] or control barrier certificates [36].

4.2 Hierarchical and distributed control in process engineering

If energy or process engineering systems are considered, the topology of the systems will change noticably compared to one of a MAS. The subsystems of these applications strongly interact with each other already without any futher coupling controls, e.g. by the exchange of energy or mass streams, while the subsystems of a MAS do not interact without any control. Thus, the objective of the control problem also changes. Furthermore, dynamics of these systems cannot be discribed by simple linear dynamics as for MASs. The dynamics are normally characterized by large nonlinear descriptor systems.

The subsystems are often controlled using optimization-based control methods such as MPC, but the controllers are usually implemented in a decentralized manner today, i.e. without accounting for interactions between the subsystems. Hence, the goal of coordinated and cooperative control focuses on how to expand existing control structures, such that the optimal solution of the global problem can be achieved. Although normally these systems can be described as coupled nonlinear ODE ore DAE systems, the main research focus is attached to linear time-invariant systems, either in continuous time or in discrete time description.

While in process systems engineering the focus in hierarchical and distributed control is on MPC, there are also other approches. E.g. Sun et al. [37] discuss a control topology based on linear state-feedback for control of distributed (and possibly renewable) energy resources.

4.2.1 Linear HD-MPC methods

Wakasa et al. apply the dual decomposition method of Lasdon to linear time-invariant systems. As the objective function of the dual problem there is non nondifferentiable, the dual problem is solved using a subgradient optimization algorithm [38]. They apply the proposed method in two case studies with 3 and 15 scalar double-ingegrator subsystems. Necoara et al. propose a dual decomposition based method, called the proximal center method, which can be applied for the optimization of linear time-invariant systems with convex objective functions [39]. Another price-driven coordination method for linear systems is suggested by Cheng et al. [9] and Marcos et al. [3],respectively. There, the coordinator updates the prices based on the gradients of given resources using Newton's method. In contrast, in dual optimization, the Lagrange multipliers are updated based on the deviation of the interaction constraints.

Delvenne et al. [40] derived a value for the worst-case performance of any distributed control strategy. They consider LTI discete-time systems with an identity input matrix. Thus the considered class of systems is quite restrictive. Vaccarini et al. [4] consider an unconstrained distributed MPC solution for a class of linear time-invariant discrete-time processes. Their method guarantees closed-loop stability, while no stability constraints have to be used. The stability results can be used for the tuning of the decentralized controller. Zhang and Li also present an MPC method for linear time-invariant discrete-time systems [41]. However they only consider a class of serially connected subsystems, i.e. cascade processes. They could derive stability results in case of unconstrained control systems. Jia et al. propose a distributed MPC scheme with stability constraint (DMPC-SC) [42, 43] for linear time-invariant systems. This scheme ensures stability for a class of controllable systems. The method is successfully applied to the load frequency control problem of a power network. Richards and How [44] propose a distributed MPC method for LTI discrete time systems. The methed is proven to guarantee robust constraint satisfaction and convergence under the assumption of bounded disturbances.

Rawlings et al. also consider linear time-invariant discrete-time systems [30, 45, 46]. However, they propose a feasible cooperation-based distributed model predictive control method, thus the iteration can be terminated at any time in order to get a feasible, but suboptimal solution. Global optimality is achieved for full convergence. Thereby local controllers require knowledge on the full global objective function. Cooperation is achieved, as at each iteration, the trajectory of inputs is a convex combination of the current local solution and the previous iteration. Performance of the coordination-based method is compared to the performace of fully centralized MPC, fully decentralized MPC and communication-based MPC. In communication-based MPC, communication of variables is assumed, though that method does not calculate convex combinations of the optimal inputs. The cooperation-based MPC is extended by Pannocchia et al. using Partial Enumeration [5]. A solution table is used to increase the speed of computations. For this purpose the solution table stores some of the most recently optimal active sets.

Negenborn combines methods of multi-agent control theory and methods of model predictive control [47, 48]. The thesis focuses on methods for linear time-invariant systems, though also non-linear optimization methods such as pattern search are applied. Thereby different control topologies, single-layer and multi-layer topolgies, are considered. The methods are exemplarily applied to various problems from the domain of power networks.

Aske et al. propose a practical coordinator MPC [49], that fits to the specific problem of a large-scale gas plant, namely the Kårstø gas processing plant in Norway. The hierarchical MPC structure consists of several local model predictive controllers and a coordinator, that controls the variables, which are economically relevant. The decomposed MPC structure is implemented using the standard in-house MPC software SEPTIC.

Dantzig-Wolfe decomposition [24] is still a method, which is in the interest of research, although it is restricted to linear problems. Gunnerud et al. [50] apply this method for the real-time optimization of oil production in a petroleum asset, namely the Troll west oil rim. However, this real-time optimization is only static. Cheng et al. [6] also apply the Dantzig-Wolfe decompostion method for a static optimization of large scale-systems. They regard the static set-point optimization as a an optimization of a linear process with linear objective function. The method is illustrated for a simple case-study, consisting of three static subsystems.

4.2.2 Nonlinear HD-MPC methods

Nonlinear model predictive control methods are quite rare in research. The most popular nonlinear method is still the dual optimization method [27]. Though, research for that method is mainly focused on the application to linear systems as described in the sections above. Talukdar et al. propose a cooperative distributed model predictive control method for nonlinear systems [51]. They assume that subsystems are only able to communicate and cooperate with neighboring subsystems. The method is applied to the IEEE 118 bus test case in order to study how to prevent cascading failures of that network. Another nonlinear method is proposed by Liu et al [52]: Their Lyapunov-based model predictive control method requires only one directional communication between the different control units. However, control is distributed only into two different controllers. The method is applied to a chemical process example consisting of two CSTRs and one flash tank separator. Gromov et al. [53] considered a very special type of system: The considered chromatographic batch process is descibed by nonlinear partial differential equations on the one hand and by discrete variables, which represent different configurations of that process, on the other hand. The hierarchical control appoach is split into two layers: On the lower layer, continuous optimzations of the partial differential equations are performed, while on the higher layer different configurations are considered. Dunbar [54] proposes a distributed MPC method and applies the method to a system of coupled oscillators. Magni and Scattolini [55] propose a stabilizing decentralized MPC scheme for nonlinear discrete-time systems. However, as the method does not involve any coordination or cooperation, the resulting control will be suboptimal.

Chapter 5

Conclusions

Hierarchical and distributed methods have now been studied for approximately 50 years. Based on the reviewed literature, we conclude, that the main interest for hierarchical and distributed model predictive control has been focused on methods for linear time-invariant systems with respect to quadratic objective functions in the past, where meaningful progress has been achieved. Hierarchical methods have been considered as well as distributed methods; feasible as well as infeasible methods.

However, research for methods, which can be applied to nonlinear systems, and which are necessary for economical or ecological dynamic optimization are still quite sparsely spread. Furthermore, the application of the methods to real large-scale processes has still to be done. Normally the methods have been applied only to small simple-toy problems. Then, many of the proposed hierarchical and distributed MPC methods suffer from bad performance [6], which is often worse than the performance of centralized MPC. This means that computational requirements for hierarchical and distributed MPC methods are often higher than those for centralized MPC methods. Hence, one of the main goals of HD-MPC methods has clearly not been achieved so far. Usually this problem is caused by the iterative nature of most of the proposed MPC methods, although a single iteration of a local problem can be solved much faster than the centralized global problem.

From the author's point of view, many of the ideas which have been established in the 1960s and 1970s and summrized by Lasdon [26], Mesarovic [10] and Findeisen [28] have only little or not at all been persued in subsequent research. In the author's point of view, it is worth looking into that literature again, in order to improve existing hierarchical and distributed MPC methods or even derive new HD-MPC approaches.

As the existing methods are really heterogeneous, an objective and comprehensive comparison of existing methods is almost impossible. The methods especially differ at first in the considered types of distributed systems: e.g. on the one hand linear systems and on the other hand nonlinear systems are examined; systems are considered in continuous-time and in discrete-time form; special types of systems as cascade systems as well as the most general interconnection toplogies are taken into account; some methods only fit to a special application; and the interactions of systems are described in various ways. Additionally methods can be devided into feasible and infeasible methods. There are model-coordination methods and goal-coordination methods, though the latter dominate the existing HD-MPC methods.

Furthermore, in order to judge the different methods one has to differentiate between different applications. While for steady-state control, as in centralized methods, linear methods can be applied very well, for processes, that are run in different operational states with transient changes of state, nonlinear methods are required in order to get a real economically and ecologically optimal control system. Then, the time constants of the process as well as the considered time horizon have to be considered, as these values define the requirements on computational performance of the model predictive control methods. Moreover, the topology of the real processes has to be known: An MPC method for the most general system topology, i.e. a system with a uniform coupling of all state variables, will most likely not be the optimal solution in application. In particular, the considered type of distributed systems usually consists of subsystem with a strong internal coupling. However the interaction with other subsystems will most likely be less strong.

Summarizing, it can be said, that, although hierarchical and distributed methods have been studied already half a century, research in this area still seems to be more in the beginning. There are still various open questions, which have to be solved, e.g.: What is the appropriate control structure for large-scale systems? How should different time-scales within different subsystems be considered in a distributed control topology? When do we have to consider nonlinear methods, for which purpose could linear methods be sufficient?

Due to an increasing competition on market as well as an increasing awareness of environmental problems and decresing resources, it is necessary to operate processes such as chemical and power plants always in an economical and ecological optimal manner. In order to achieve this high goal, it is necessary to apply nonlinear control methods for the operation of the considered plants. However, according to the literature, there is clear lack of nonlinear HD-MPC methods so far. Thus, it has to be one goal for the following years, to shift the focus from linear to nonlinear methods. Though, research on linear methods will still have some importance, as most of the analytic properties, such as stability, robustness, and convergence, can mainly be prooven for linear systems.

An important aspect, which has to be considered, is the fact that the developed methods will be applied only in future systems with future computer architectures. Hence, computational power will then be further increased, such that an application of nonlinear methods becomes more realistic than it is today.

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